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GCE

# Mathematics

Advanced GCE **A2 7890 – 2**

Advanced Subsidiary GCE **AS 3890 – 2**

# Mark Schemes for the Units

# June 2009

**3890-2/7890-2/MS/R/09**

Oxford Cambridge and RSA Examinations

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# 4721 Core Mathematics 1

| | | | | |
|---|---|---|---|---|
| 1 | (i) | $\frac{dy}{dx} = 5x^4 - 2x^{-3}$ | **B1**<br>**M1**<br>**A1** **3** | $5x^4$<br>$x^{-2}$ before differentiation or $kx^{-3}$ in $\frac{dy}{dx}$ soi<br>$-2x^{-3}$ |
| | (ii) | $\frac{d^2y}{dx^2} = 20x^3 + 6x^{-4}$ | **M1**<br>**A1** **2**<br>**5** | Attempt to differentiate their (i) – at least one term correct<br>cao |
| 2 | | $\frac{(8+\sqrt{7})(2-\sqrt{7})}{(2+\sqrt{7})(2-\sqrt{7})}$<br>$= \frac{9-6\sqrt{7}}{4-7}$<br>$= -3+2\sqrt{7}$ | **M1**<br>**A1**<br>**A1**<br>**A1** **4**<br>**4** | Multiply numerator and denominator by conjugate<br>Numerator correct and simplified<br>Denominator correct and simplified<br>cao |
| 3 | (i) | $3^{-2}$ | **B1** **1** | |
| | (ii) | $3^{\frac{1}{3}}$ | **B1** **1** | |
| | (iii) | $3^{10} \times 3^{30}$<br>$= 3^{40}$ | **M1**<br>**A1** **2**<br>**4** | $3^{30}$ or $9^{20}$ soi |
| 4 | | $y = 2x - 4$<br>$4x^2 + (2x-4)^2 = 10$<br>$8x^2 - 16x + 16 = 10$<br>$8x^2 - 16x + 6 = 0$<br>$4x^2 - 8x + 3 = 0$<br>$(2x-1)(2x-3) = 0$<br>$x = \frac{1}{2}$ , $x = \frac{3}{2}$<br>$y =$ -3, $y =$ -1 | **M1***<br>**A1**<br>**M1dep***<br>**A1**<br>**A1**<br>**A1** **6**<br>**6** | Attempt to get an equation in 1 variable only<br>Obtain correct 3 term quadratic (aef)<br>Correct method to solve quadratic of form $ax^2 + bx + c = 0$ $(b \neq 0)$<br>Correct factorisation oe<br>Both x values correct<br>Both y values correct<br>or<br>one correct pair of values www B1<br>second correct pair of values B1 |

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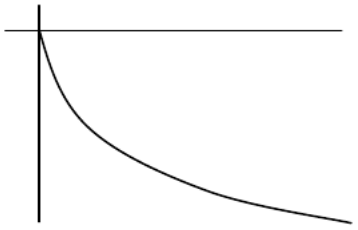
|---|---|---|---|---|
| 5 | (i) | $(2x^2-5x-3)(x+4)$<br>$=2x^3+8x^2-5x^2-20x-3x-12$<br>$=2x^3+3x^2-23x-12$ | M1<br><br>A1<br>A1 3 | Attempt to multiply a quadratic by a linear factor or to expand all 3 brackets with an appropriate number of terms (including an $x^3$ term)<br>Expansion with no more than one incorrect term |
| | (ii) | $2x^4+7x^4$<br>$=9x^4$<br>9 | B1<br>B1 2<br><br>5 | $2x^4$ or $7x^4$ soi www<br>$9x^4$ or 9 |
| 6 | (i) |  | B1<br><br>B1 2 | One to one graph only in bottom right hand quadrant<br>Correct graph, passing through origin |
| | (ii) | Translation<br>Parallel to $y$-axis, 5 units | B1<br>B1 2 | |
| | (iii) | $y=-\sqrt{\frac{x}{2}}$ | M1<br><br>A1 2<br>6 | $\sqrt{2x}$ or $\sqrt{\frac{x}{2}}$ seen<br>cao |
| 7 | (i) | $\left(x-\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2+\frac{1}{4}$<br>$=\left(x-\frac{5}{2}\right)^2-6$ | B1<br><br>M1<br><br>A1 3 | $a=\frac{5}{2}$<br>$\frac{1}{4}-a^2$<br>cao |
| | (ii) | $\left(x-\frac{5}{2}\right)^2-6+y^2=0$<br>Centre $\left(\frac{5}{2},0\right)$<br><br>Radius = $\sqrt{6}$ | <br><br>B1<br>B1<br><br>B1 3<br>6 | <br><br>Correct $x$ coordinate<br>Correct $y$ coordinate<br> |

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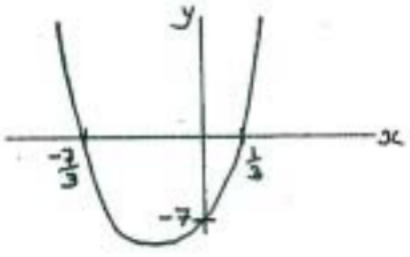
|---|---|---|---|---|
| **8** | **(i)** | $-42 < 6x < -6$ | **M1** | 2 equations or inequalities both dealing with all 3 terms |
| | | $-7 < x < -1$ | **A1** | -7 and -1 seen oe |
| | | | **A1 3** | $-7 < x < -1$ (or $x > -7$ and $x < -1$) |
| | **(ii)** | $x^2 > 16$ | **B1** | ±4 oe seen |
| | | $x > 4$ | **B1** | $x > 4$ |
| | | or $x < -4$ | **B1 3** | $x < -4$ not wrapped, not 'and' |
| | | | **6** | |
| **9** | **(i)** | $\sqrt{(^-1-4)^2+(9-^-3)^2}$ | **M1** | Correct method to find line length using Pythagoras' theorem |
| | | $=13$ | **A1 2** | cao |
| | **(ii)** | $\left(\frac{4+^-1}{2}, \frac{^-3+9}{2}\right)$ | **M1** | Correct method to find midpoint |
| | | $\left(\frac{3}{2}, 3\right)$ | **A1 2** | |
| | **(iii)** | Gradient of $AB = -\frac{12}{5}$ | **B1** | |
| | | $y-3=-\frac{12}{5}(x-1)$ | **M1** | Correct equation for line, any gradient, through (1, 3) |
| | | $12x+5y-27=0$ | **A1** | Correct equation in any form with gradient simplified |
| | | | **A1 4** | $12x+5y-27=0$ |
| | | | **8** | |
| **10** | **(i)** | $(3x+7)(3x-1)=0$ | **M1** | Correct method to find roots |
| | | $x=-\frac{7}{3}, x=\frac{1}{3}$ | **A1** | Correct factorisation oe |
| | | | **A1 3** | Correct roots |
| | **(ii)** | $\frac{dy}{dx}=18x+18$ | **M1** | Attempt to differentiate $y$ |
| | | $18x+18=0$ | **M1** | Uses $\frac{dy}{dx}=0$ |
| | | $x=-1$ | **A1** | |
| | | $y=-16$ | **A1 ft 4** | |
| | **(iii)** |   | **B1** | Positive quadratic curve |
| | | | **B1** | $y$ intercept (0, -7) |
| | | | **B1 3** | Good graph, with correct roots indicated and minimum point in correct quadrant |
| | **(iv)** | $x > -1$ | **B1 1** | |
| | | | **11** | |

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| | | | |
|---|---|---|---|
| **11 (i)** | Gradient of normal = $-\frac{2}{3}$ | **B1** | |
| | $\frac{dy}{dx} = \frac{1}{2}kx^{\frac{1}{2}}$ | **M1*** | Attempt to differentiate equation of curve |
| | | **A1** | $\frac{1}{2}kx^{-\frac{1}{2}}$ |
| | When $x = 4$, $\frac{dy}{dx} = \frac{k}{4}$ | **M1dep*** | Attempt to substitute $x = 4$ into their $\frac{dy}{dx}$ soi |
| | $\therefore \frac{k}{4} = \frac{3}{2}$ | **M1dep*** | Equate their gradient expression to negative reciprocal of their gradient of normal |
| | $k = 6$ | **A1 6** | cao |
| **(ii)** | $P$ is point (4, 12) | **B1 ft** | |
| | $Q$ is point (22, 0) | **M1** | Correct method to find coordinates of $Q$ |
| | | **A1** | Correct $x$ coordinate |
| | Area of triangle $= \frac{1}{2} \times 12 \times 22$ | **M1** | Must use $y$ coordinate of P and $x$ coordinate of Q |
| | $= 132$ sq. units | **A1 5** **11** | |

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# 4722 Core Mathematics 2

| | | | | |
|---|---|---|---|---|
| **1** | **(i)** $\cos\theta = \frac{6.4^2+7.0^2-11.3^2}{2\times6.4\times7.0}$ | M1 | | Attempt use of cosine rule (any angle) |
| | $= -0.4211$ | A1 | | Obtain one of $115^o$, $34.2^o$, $30.9^o$, 2.01, 0.597, 0.539 |
| | $\theta = 115^o$ or 2.01 rads | A1 | **3** | Obtain $115^o$ or 2.01 rads, or better |
| | **(ii)** area $= \frac{1}{2}\times7\times6.4\times\sin115$ | M1 | | Attempt triangle area using (½)*ab*sin*C*, or equiv |
| | $= 20.3$ cm$^2$ | A1 | **2** | Obtain 20.3 (cao) |
| | | | **5** | |
| **2** | **(i)** $a + 9d = 2(a + 3d)$ | M1* | | Attempt use of $a + (n-1)d$ or $a + nd$ at least once for $u_4$, $u_{10}$ or $u_{20}$ |
| | $a = 3d$ | A1 | | Obtain $a = 3d$ (or unsimplified equiv) and $a + 19d = 44$ |
| | $a + 19d = 44 \Rightarrow 22d = 44$ | M1dep* | | Attempt to eliminate one variable from two simultaneous equations in $a$ and $d$, from $u_4$, $u_{10}$, $u_{20}$ and no others |
| | $d = 2$, $a = 6$ | A1 | **4** | Obtain $d = 2$, $a = 6$ |
| | **(ii)** $S_{50} = {}^{50}/_{2}\ (2x6 + 49x2)$ | M1 | | Attempt $S_{50}$ of AP, using correct formula, with $n = 50$, allow $25(2a + 24d)$ |
| | $= 2750$ | A1 | **2** | Obtain 2750 |
| | | | **6** | |
| **3** | $\log 7^x = \log 2^{x+1}$ | M1 | | Introduce logarithms throughout, or equiv with base 7 or 2 |
| | $x\log 7 = (x+1)\log 2$ | M1 | | Drop power on at least one side |
| | | A1 | | Obtain correct linear equation (allow with no brackets) |
| | $x(\log 7 - \log 2) = \log 2$ | M1 | | **Either** expand bracket and attempt to gather $x$ terms, **or** deal correctly with algebraic fraction |
| | $x = 0.553$ | A1 | **5** | Obtain $x = 0.55$, or rounding to this, with no errors seen |
| | | | **5** | |
| **4** | **(i)** $(x^2-5)^3 = (x^2)^3 + 3(x^2)^2(-5) + 3(x^2)(-5)^2 + (-5)^3$ | M1* | | Attempt expansion, with product of powers of $x^2$ and $\pm 5$, at least 3 terms |
| | $= x^6 - 15x^4 + 75x^2 - 125$ | M1* | | Use at least 3 of binomial coeffs of 1, 3, 3, 1 |
| | | A1dep* | | Obtain at least two correct terms, coeffs simplified |
| | | A1 | **4** | Obtain fully correct expansion, coeffs simplified |
| | *OR* | | | |
| | $(x^2-5)^3 = (x^2-5)(x^4-10x^2+25)$ | M2 | | Attempt full expansion of all 3 brackets |
| | $= x^6 - 15x^4 + 75x^2 - 125$ | A1 | | Obtain at least two correct terms |
| | | A1 | | Obtain full correct expansion |
| | **(ii)** $\int (x^2-5)^3\,dx = \frac{1}{7}x^7 - 3x^5 + 25x^3 - 125x + c$ | M1 | | Attempt integration of terms of form $kx^n$ |
| | | A1√ | | Obtain at least two correct terms, allow unsimplified coeffs |
| | | A1 | | Obtain $\frac{1}{7}x^7 - 3x^5 + 25x^3 - 125x$ |
| | | B1 | **4** | $+ c$, and no d$x$ or $\int$ sign |
| | | | **8** | |

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|---|---|---|---|---|
| **5** **(i)** | $2x = 30^o, 150^o$ | M1 | | Attempt $\sin^{-1}$ 0.5, then divide or multiply by 2 |
| | $x = 15^o, 75^o$ | A1 | | Obtain $15^o$ (allow $\pi/12$ or 0.262) |
| | | A1 | **3** | Obtain $75^o$ (not radians), and no extra solutions in range |
| **(ii)** | $2(1 - \cos^2 x) = 2 - \sqrt{3}\cos x$ | M1 | | Use $\sin^2 x = 1 - \cos^2 x$ |
| | $2\cos^2 x - \sqrt{3}\cos x = 0$ | A1 | | Obtain $2\cos^2 x - \sqrt{3}\cos x = 0$ or equiv (no constant terms) |
| | $\cos x\,(2\cos x - \sqrt{3}) = 0$ | M1 | | Attempt to solve quadratic in $\cos x$ |
| | $\cos x = 0$, $\cos x = \frac{1}{2}\sqrt{3}$ | A1 | | Obtain $30^o$ (allow $\pi/6$ or 0524), and no extra solns in range |
| | $x = 90^o$ , $x = 30^o$ | B1 | **5** | Obtain $90^o$ (allow $\pi/2$ or 1.57), from correct quadratic only |
| | | **SR** | | answer only B1 one correct solution |
| | | | | B1 second correct solution, and no others |
| | | | **8** | |
| **6** | $\int (3x^2 + a)\,dx = x^3 + ax + c$ | M1 | | Attempt to integrate |
| | | A1 | | Obtain at least one correct term, allow unsimplified |
| | | A1 | | Obtain $x^3 + ax$ |
| | $(-1, 2) \Rightarrow -1 - a + c = 2$ | M1 | | Substitute at least one of $(-1, 2)$ or $(2, 17)$ into integration attempt involving $a$ and $c$ |
| | $(2, 17) \Rightarrow 8 + 2a + c = 17$ | A1 | | Obtain two correct equations, allow unsimplified |
| | | M1 | | Attempt to eliminate one variable from two equations in $a$ and $c$ |
| | $a = 2$, $c = 5$ | A1 | | Obtain $a = 2$, $c = 5$, from correct equations |
| | Hence $y = x^3 + 2x + 5$ | A1 | **8** | State $y = x^3 + 2x + 5$ |
| | | | **8** | |
| **7** **(i)** | f(-2) = -16 + 36 – 22 – 8 | M1 | | Attempt f(-2), or equiv |
| | = -10 | A1 | **2** | Obtain -10 |
| **(ii)** | f(½) = ¼ + 2¼ + 5½ – 8 = 0 AG | M1 | | Attempt f(½) (no other method allowed) |
| | | A1 | **2** | Confirm f(½) = 0, extra line of working required |
| **(iii)** | $f(x) = (2x - 1)(x^2 + 5x + 8)$ | M1 | | Attempt complete division by $(2x - 1)$ or $(x - \frac{1}{2})$ or equiv |
| | | A1 | | Obtain $x^2 + 5x + c$ or $2x^2 + 10x + c$ |
| | | A1 | **3** | State $(2x - 1)(x^2 + 5x + 8)$ or $(x - \frac{1}{2})(2x^2 + 10x + 16)$ |
| **(iv)** | $f(x)$ has one real root ($x = \frac{1}{2}$) | B1√ | | State 1 root, following their quotient, ignore reason |
| | because $b^2 - 4ac = 25 - 32 = -7$ | | | |
| | hence quadratic has no real roots as $-7 < 0$, | B1√ | **2** | Correct calculation, eg discriminant or quadratic formula, following their quotient, or cubic has max at (-2.15, -9.9) |
| | | | **9** | |

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|---|---|---|---|---|
| **8** **(i)** | $\frac{1}{2} \times r^2 \times 1.2 = 60$ | M1 | | Attempt $(\frac{1}{2}) r^2\theta = 60$ |
| | $r = 10$ | A1 | | Obtain $r = 10$ |
| | $r\theta = 10 \times 1.2 = 12$ | B1√ | | State or imply arc length is $1.2r$, following their $r$ |
| | perimeter = 10 + 10 + 12 = 32 cm | A1 | **4** | Obtain 32 |
| **(ii)(a)** | $u_5 = 60 \times 0.6^4$ | M1 | | Attempt $u_5$ using $ar^4$, or list terms |
| | $= 7.78$ | A1 | **2** | Obtain 7.78, or better |
| **(b)** | $S_{10} = \frac{60(1-0.6^{10})}{1-0.6}$ | M1 | | Attempt use of correct sum formula for a GP, or sum terms |
| | $= 149$ | A1 | **2** | Obtain 149, or better (allow 149.0 – 149.2 inclusive) |
| **(c)** | common ratio is less than 1, so series is convergent and hence sum to infinity exists | B1 | | series is convergent or $-1 < r < 1$ (allow $r < 1$) or reference to areas getting smaller / adding on less each time |
| | $S_\infty = \frac{60}{1-0.6}$ | M1 | | Attempt $S_\infty$ using $\frac{a}{1-r}$ |
| | $= 150$ | A1 | **3** | Obtain $S_\infty = 150$ |
| | | | | **SR** B1 only for 150 with no method shown |
| | | | **11** | |
| **9** **(i)** | | B1 | | Sketch graph showing exponential growth (both quadrants) |
| | | B1 | **2** | State or imply (0, 4) |
| **(ii)** | $4k^x = 20k^2$ | | | |
| | $k^x = 5k^2$ | M1 | | Equate $4k^x$ to $20k^2$ and take logs (any, or no, base) |
| | $x = \log_k 5k^2$ | | | |
| | $x = \log_k 5 + \log_k k^2$ | M1 | | Use $\log ab = \log a + \log b$ |
| | $x = 2\log_k k + \log_k 5$ | M1 | | Use $\log a^b = b \log a$ |
| | $x = 2 + \log_k 5$ AG | A1 | **4** | Show given answer correctly |
| *OR* | $4k^x = 20k^2$ | | | |
| | $k^x = 5k^2$ | M1 | | Attempt to rewrite as single index |
| | $k^{x-2} = 5$ | A1 | | Obtain $k^{x-2} = 5$ or equiv eg $4k^{x-2} = 20$ |
| | $x - 2 = \log_k 5$ | M1 | | Take logs (to any base) |
| | $x = 2 + \log_k 5$ AG | A1 | | Show given answer correctly |
| **(iii)** **(a)** | area $\approx \frac{1}{2} \times \frac{1}{2} \times \left(4k^0 + 8k^{\frac{1}{2}} + 4k^1\right)$ | M1 | | Attempt $y$-values at $x = 0$, ½ and 1, and no others |
| | | M1 | | Attempt to use correct trapezium rule, 3 $y$-values, $h = ½$ |
| | $\approx 1 + 2k^{\frac{1}{2}} + k$ | A1 | **3** | Obtain a correct expression, allow unsimplified |
| **(b)** | $1 + 2k^{\frac{1}{2}} + k = 16$ | M1 | | Equate attempt at area to 16 |
| | $\left(k^{\frac{1}{2}} + 1\right)^2 = 16$ | M1 | | Attempt to solve 'disguised' 3 term quadratic |
| | $k^{\frac{1}{2}} = 3$ | | | |
| | $k = 9$ | A1 | **3** | Obtain $k = 9$ only |
| | | | **12** | |

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# 4723 Core Mathematics 3

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|---|---|---|---|---|
| **1 (i)** | State $y = \sec x$ | B1 | | |
| **(ii)** | State $y = \cot x$ | B1 | | |
| **(iii)** | State $y = \sin^{-1} x$ | B1 | 3 | |
| | | | **3** | |

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| | | | | |
|---|---|---|---|---|
| **2** | Either: State or imply $\int \pi(2x-3)^4 \, dx$ | B1 | | or unsimplified equiv |
| | Obtain integral of form $k(2x-3)^5$ | M1 | | any constant $k$ involving $\pi$ or not |
| | Obtain $\frac{1}{10}(2x-3)^5$ or $\frac{1}{10}\pi(2x-3)^5$ | A1 | | |
| | Attempt evaluation using 0 and $\frac{3}{2}$ | M1 | | subtraction correct way round |
| | Obtain $\frac{243}{10}\pi$ | A1 | 5 | or exact equiv |
| | Or: State or imply $\int \pi(2x-3)^4 \, dx$ | B1 | | or unsimplified equiv |
| | Expand and obtain integral of order 5 | M1 | | with at least three terms correct |
| | Ob'n $\frac{16}{5}x^5 - 24x^4 + 72x^3 - 108x^2 + 81x$ | A1 | | with or without $\pi$ |
| | Attempt evaluation using (0 and) $\frac{3}{2}$ | M1 | | |
| | Obtain $\frac{243}{10}\pi$ | A1 | (5) | or exact equiv |
| | | | **5** | |

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|---|---|---|---|---|
| **3 (i)** | Attempt use of identity for $\sec^2 \alpha$ | M1 | | using $\pm\tan^2\alpha \pm 1$ |
| | Obtain $1 + (m+2)^2 - (1+m^2)$ | A1 | | absent brackets implied by subsequent correct working |
| | Obtain $4m + 4 = 16$ and hence $m = 3$ | A1 | 3 | |
| **(ii)** | Attempt subn in identity for $\tan(\alpha + \beta)$ | M1 | | using $\frac{\pm\tan\alpha \pm \tan\beta}{1 \pm \tan\alpha\tan\beta}$ |
| | Obtain $\frac{5+3}{1-15}$ or $\frac{m+2+m}{1-m(m+2)}$ | A1√ | | following their $m$ |
| | Obtain $-\frac{4}{7}$ | A1 | 3 | or exact equiv |
| | | | **6** | |

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| | | | | |
|---|---|---|---|---|
| **4 (i)** | Obtain $\frac{1}{3}e^{3x} + e^x$ | B1 | | |
| | Substitute to obtain $\frac{1}{3}e^{9a} + e^{3a} - \frac{1}{3}e^{3a} - e^a$ | B1 | | or equiv |
| | Equate definite integral to 100 and attempt rearrangement | M1 | | as far as $e^{9a} = ...$ |
| | Introduce natural logarithm | M1 | | using correct process |
| | Obtain $a = \frac{1}{9}\ln(300 + 3e^a - 2e^{3a})$ | A1 | 5 | AG; necessary detail needed |
| **(ii)** | Obtain correct first iterate | B1 | | allow for 4 dp rounded or truncated |
| | Show correct iteration process | M1 | | with at least one more step |
| | Obtain at least three correct iterates in all | A1 | | allowing recovery after error |
| | Obtain 0.6309 | A1 | 4 | following at least three correct steps; answer required to exactly 4 dp |
| | [0.6 → 0.631269 → 0.630884 → 0.630889] | | | |
| | | | **9** | |

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| **5 (i)** | Either: Show correct process for comp'n | M1 | | correct way round and in terms of $x$ |
| | Obtain $y = 3(3x+7)-2$ | A1 | | or equiv |
| | Obtain $x = -\frac{19}{9}$ | A1 | 3 | or exact equiv; condone absence of $y = 0$ |
| | Or: Use $\text{fg}(x) = 0$ to obtain $\text{g}(x) = \frac{2}{3}$ | B1 | | |
| | Attempt solution of $\text{g}(x) = \frac{2}{3}$ | M1 | | |
| | Obtain $x = -\frac{19}{9}$ | A1 | (3) | or exact equiv; condone absence of $y = 0$ |
| **(ii)** | Attempt formation of one of the equations $3x+7 = \frac{x-7}{3}$ or $3x+7 = x$ or $\frac{x-7}{3} = x$ | M1 | | or equiv |
| | Obtain $x = -\frac{7}{2}$ | A1 | | or equiv |
| | Obtain $y = -\frac{7}{2}$ | A1√ | 3 | or equiv; following their value of $x$ |
| **(iii)** | Attempt solution of modulus equation | M1 | | squaring both sides to obtain 3-term quadratics or forming linear equation with signs of $3x$ different on each side |
| | Obtain $-12x+4 = 42x+49$ or $3x-2 = -3x-7$ | A1 | | or equiv |
| | Obtain $x = -\frac{5}{6}$ | A1 | | or exact equiv; as final answer |
| | Obtain $y = \frac{9}{2}$ | A1 | 4 | or equiv; and no other pair of answers |
| | | | **10** | |

| | | | | |
|---|---|---|---|---|
| **6 (i)** | Obtain derivative $k(37+10y-2y^2)^{-\frac{1}{2}}\text{f}(y)$ | M1 | | any constant $k$; any linear function for f |
| | Obtain $\frac{1}{2}(10-4y)(37+10y-2y^2)^{-\frac{1}{2}}$ | A1 | 2 | or equiv |
| **(ii)** | Either: Sub'te $y = 3$ in expression for $\frac{dx}{dy}$ | *M1 | | |
| | Take reciprocal of expression/value | *M1 | | and without change of sign |
| | Obtain $-7$ for gradient of tangent | A1 | | |
| | Attempt equation of tangent | M1 | | dep *M *M |
| | Obtain $y = -7x+52$ | A1 | 5 | and no second equation |
| | Or: Sub'te $y = 3$ in expression for $\frac{dx}{dy}$ | M1 | | |
| | Attempt formation of eq'n $x = m'y+c$ | M1 | | where $m'$ is attempt at $\frac{dx}{dy}$ |
| | Obtain $x-7 = -\frac{1}{7}(y-3)$ | A1 | | or equiv |
| | Attempt rearrangement to required form | M1 | | |
| | Obtain $y = -7x+52$ | A1 | (5) | and no second equation |
| | | | **7** | |

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| | | | | |
|---|---|---|---|---|
| **7 (i)** | State $R = 10$ | B1 | | or equiv |
| | Attempt to find value of $\alpha$ | M1 | | implied by correct answer or its complement; allow sin/cos muddles |
| | Obtain 36.9 or $\tan^{-1}\frac{3}{4}$ | A1 | 3 | or greater accuracy 36.8699… |
| **(ii)(a)** | Show correct process for finding one angle | M1 | | |
| | Obtain (64.16 + 36.87 and hence) 101 | A1 | | or greater accuracy 101.027… |
| | Show correct process for finding second angle | M1 | | |
| | Obtain (115.84 + 36.87 and hence) 153 | A1√ | 4 | following their value of $\alpha$ ; or greater accuracy 152.711…; and no other between 0 and 360 |
| **(b)** | Recognise link with part **(i)** | M1 | | signalled by 40 … – 20 … |
| | Use fact that maximum and minimum values of sine are 1 and –1 | M1 | | may be implied; or equiv |
| | Obtain 60 | A1 | 3 | |
| | | | **10** | |
| **8 (i)** | Refer to translation and stretch | M1 | | in either order; allow here equiv informal terms such as 'move', … |
| | State translation in $x$ direction by 6 | A1 | | or equiv; now with correct terminology |
| | State stretch in $y$ direction by 2 | A1 | 3 | or equiv; now with correct terminology |
| | [SC: if M0 but one transformation completely correct, give B1] | | | |
| **(ii)** | State $2\ln(x-6) = \ln x$ | B1 | | or $2\ln(a-6) = \ln a$ or equiv |
| | Show correct use of logarithm property | *M1 | | |
| | Attempt solution of 3-term quadratic | M1 | | dep *M |
| | Obtain 9 only | A1 | 4 | following correct solution of equation |
| **(iii)** | Attempt evaluation of form $k(y_0 + 4y_1 + y_2)$ | M1 | | any constant $k$; maybe with $y_0 = 0$ implied |
| | Obtain $\frac{1}{3}\times 1(2\ln 1 + 8\ln 2 + 2\ln 3)$ | A1 | | or equiv |
| | Obtain 2.58 | A1 | 3 | or greater accuracy 2.5808… |
| | | | **10** | |
| **9 (a)** | Attempt use of quotient rule | *M1 | | or equiv; allow numerator wrong way round and denominator errors |
| | Obtain $\dfrac{(kx^2+1)2kx - (kx^2-1)2kx}{(kx^2+1)^2}$ | A1 | | or equiv; with absent brackets implied by subsequent correct working |
| | Obtain correct simplified numerator $4kx$ | A1 | | |
| | Equate numerator of first derivative to zero | M1 | | dep *M |
| | State $x = 0$ or refer to $4kx$ being linear or observe that, with $k \neq 0$, only one sol'n | A1√ | 5 | AG or equiv; following numerator of form $k'kx = 0$, any constant $k'$ |

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|---|---|---|---|---|
| **(b)** | Attempt use of product rule | *M1 | | |
| | Obtain $me^{mx}(x^2+mx)+e^{mx}(2x+m)$ | A1 | | or equiv |
| | Equate to zero and either factorise with factor $e^{mx}$ or divide through by $e^{mx}$ | M1 | | dep *M |
| | Obtain $mx^2+(m^2+2)x+m=0$ or equiv and observe that $e^{mx}$ cannot be zero | A1 | | |
| | Attempt use of discriminant | M1 | | using correct $b^2-4ac$ with their $a$, $b$, $c$ |
| | Simplify to obtain $m^4+4$ | A1 | | or equiv |
| | Observe that this is positive for all $m$ and hence two roots | A1 | 7 | or equiv; AG |
| | | | **12** | |

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# 4724 Core Mathematics 4

**1** Long Division For leading term $3x^2$ in quotient B1

Suff evid of div process ( $ax^2$ , mult back, attempt sub) M1

(Quotient) = $3x^2 - 4x - 5$ A1

(Remainder) = $-x + 2$ A1

Identity $3x^4 - x^3 - 3x^2 - 14x - 8 = Q(x^2 + x + 2) + R$ *M1

$Q = ax^2 + bx + c, R = dx + e$ & attempt $\geq 3$ ops. dep*M1 If $a = 3$ ,this $\Rightarrow$ 1 operation

$a = 3, b = -4, c = -5$ A1 dep*M1; $Q = ax^2 + bx + c$

$d = -1, e = 2$ A1

Inspection Use 'Identity' method; if $R = e$, check cf($x$) correct before awarding 2nd M1

**4**

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**2** Indefinite Integral Attempt to connect $dx$ & $d\theta$ *M1 Incl $\frac{dx}{d\theta}$ or $\frac{d\theta}{dx}$ ; not $dx = d\theta$

Reduce to $\int 1 - \tan^2\theta \, (d\theta)$ A1 A0 if $\frac{d\theta}{dx} = \sec^2\theta$ ; but allow all following A marks

Use $\tan^2\theta = (1,-1) + (\sec^2\theta, -\sec^2\theta)$ dep*M1

Produce $\int 2 - \sec^2\theta \, (d\theta)$ A1

Correct √ integration of function of type $d + e\sec^2\theta$ √A1 including $d = 0$

EITHER Attempt limits change (allow degrees here) M1 (This is 'limits' aspect; the

OR Attempt integ, re-subst & use original $(\sqrt{3}, 1)$ integ need not be accurate)

$\frac{1}{6}\pi - \sqrt{3} + 1$ isw Exact answer required A1

**7**

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| | | | | |
|---|---|---|---|---|
| **3** | **(i)** | $\left(1+\frac{x}{a}\right)^{-2} = 1+(-2)\frac{x}{a}+\frac{-2,-3}{2}\left(\frac{x}{a}\right)^2+\ldots$ | M1 | Check 3rd term; accept $\frac{x^2}{a}$ |
| | | $= 1-\frac{2x}{a}+\ldots$ or $1+\left(-\frac{2x}{a}\right)$ | B1 | or $1-2xa^{-1}$ (Ind of M1) |
| | | $\ldots + \frac{3x^2}{a^2}+\ldots$ (or $3\left(\frac{x}{a}\right)^2$ or $3x^2a^{-2}$) | A1 | Accept $\frac{6}{2}$ for 3 |
| | | $(a+x)^{-2} = \frac{1}{a^2}\{$their expansion of $\left(1+\frac{x}{a}\right)^{-2}\}$ mult out | √A1 **4** | $\frac{1}{a^2}-\frac{2x}{a^3}+\frac{3x^2}{a^4}$ ; accept eg $a^{-2}$ |
| | **(ii)** | Mult out $(1-x)$(their exp) to produce all terms/cfs($x^2$) | M1 | Ignore other terms |
| | | Produce $\frac{3}{a^2}+\frac{2}{a}(=0)$ or $\frac{3}{a^4}+\frac{2}{a^3}(=0)$ or AEF | A1 | Accept $x^2$ if in both terms |
| | | $a=-\frac{3}{2}$ www seen anywhere in (i) or (ii) | A1 **3** | Disregard any ref to $a=0$ |
| | | | **7** | |
| **4** | **(i)** | Differentiate as a product, $u\,\mathrm{d}v+v\,\mathrm{d}u$ | M1 | or as 2 separate products |
| | | $\frac{\mathrm{d}}{\mathrm{d}x}(\sin 2x)=2\cos 2x$ or $\frac{\mathrm{d}}{\mathrm{d}x}(\cos 2x)=-2\sin 2x$ | B1 | |
| | | $\mathrm{e}^x(2\cos 2x+4\sin 2x)+\mathrm{e}^x(\sin 2x-2\cos 2x)$ | A1 | terms may be in diff order |
| | | Simplify to $5\,\mathrm{e}^x\sin 2x$ www | A1 **4** | Accept $10\mathrm{e}^x\sin x\cos x$ |
| | **(ii)** | Provided result (i) is of form $k\,\mathrm{e}^x\sin 2x$, $k$ const | | |
| | | $\int \mathrm{e}^x\sin 2x\,\mathrm{d}x = \frac{1}{k}\mathrm{e}^x(\sin 2x-2\cos 2x)$ | B1 | |
| | | $[\mathrm{e}^x(\sin 2x-2\cos 2x)]_0^{\frac{1}{4}\pi} = \mathrm{e}^{\frac{1}{4}\pi}+2$ | B1 | |
| | | $\frac{1}{5}\left(\mathrm{e}^{\frac{1}{4}\pi}+2\right)$ | B1 **3** | Exact form to be seen |
| | | **SR** Although 'Hence', award M2 for double integration by parts and solving + A1 for correct answer. | | |
| | | | **7** | |

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**5 (i)** $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ aef used M1

$= \frac{4t + 3t^2}{2 + 2t}$ A1

Attempt to find $t$ from one/both equations M1 or diff (ii) cartesian eqn → M1

State/imply $t = -3$ is only solution of both equations A1 subst $(3, -9)$, solve for $\frac{dy}{dx}$ → M1

Gradient of curve = $-\frac{15}{4}$ or $\frac{-15}{4}$ or $\frac{15}{-4}$ A1 **5** grad of curve = $-\frac{15}{4}$ → A1

[**SR** If $t = 1$ is given as solution & not disqualified, award A0 + √A1 for grad = $-\frac{15}{4}$ & $\frac{7}{4}$;

If $t = 1$ is given/used as only solution, award A0 + √A1 for grad = $\frac{7}{4}$ ]

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**(ii)** $\frac{y}{x} = t$ B1

Substitute into either parametric eqn M1

Final answer $x^3 = 2xy + y^2$ A2 **4**

[**SR** Any correct unsimplified form (involving fractions or common factors) → A1]

**9**

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**6 (i)** $4x \equiv A(x-3)^2 + B(x-3)(x-5) + C(x-5)$ M1

$A = 5$ A1 'cover-up' rule, award B1

$B = -5$ A1

$C = -6$ A1 **4** 'cover-up' rule, award B1

Cands adopting other alg. manip. may be awarded M1 for a full satis method + 3 @ A1

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**(ii)** $\int \frac{A}{x-5}\,dx = A\ln(5-x)$ or $A\ln|5-x|$ or $A\ln|x-5|$ √B1 but not $A\ln(x-5)$

$\int \frac{B}{x-3}\,dx = B\ln(3-x)$ or $B\ln|3-x|$ or $B\ln|x-3|$ √B1 but not $B\ln(x-3)$

If candidate is awarded B0,B0, then award **SR** √ B1 for $A\ln(x-5)$ **and** $B\ln(x-3)$

$\int \frac{C}{(x-3)^2}\,dx = -\frac{C}{x-3}$ √B1

$5\ln\frac{3}{4} + 5\ln 2$ aef, isw $\sqrt{}\ A\ln\frac{3}{4} - B\ln 2$ √ B1 Allow if **SR** B1 awarded

$-3$ $\sqrt{}\ \frac{1}{2}C$ √B1 **5**

[Mark at earliest correct stage & isw; no ln 1] **9**

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| | | | | | |
|---|---|---|---|---|---|
| 7 | (i) | Attempt scalar prod {**u**.(4**i** + **k**) or **u**.(4**i** + 3**j** + 2**k**)}= 0 | M1 | | where **u** is the given vector |
| | | Obtain $\frac{12}{13}+c=0$ or $\frac{12}{13}+3b+2c=0$ | A1 | | |
| | | $c=-\frac{12}{13}$ | A1 | | |
| | | $b=\frac{4}{13}$ | A1 | | cao No ft |
| | | Evaluate $\left(\frac{3}{13}\right)^2+(\text{their } b)^2+(\text{their } c)^2$ | M1 | | Ignore non-mention of $\sqrt{}$ |
| | | Obtain $\frac{9}{169}+\frac{144}{169}+\frac{16}{169}=1$ AG | A1 | **6** | Ignore non-mention of $\sqrt{}$ |
| | (ii) | Use $\cos\theta=\frac{\mathbf{x}.\mathbf{y}}{\lvert\mathbf{x}\rvert\lvert\mathbf{y}\rvert}$ | M1 | | |
| | | Correct method for finding scalar product | M1 | | |
| | | 36° (35.837653…) Accept 0.625 (rad) | A1 | **3** | From $\frac{18}{\sqrt{17}\sqrt{29}}$ |
| | | **SR** If 4**i**+**k** = (4,1,0) in (i) & (ii), mark as scheme but allow final A1 for 31°(31.160968) or 0.544 | | | |
| | | | | **9** | |
| 8 | (i) | $\frac{d}{dx}(y^2)=2y\frac{dy}{dx}$ | B1 | | |
| | | $\frac{d}{dx}(uv)=u\,dv+v\,du$ used on $(-7)xy$ | M1 | | |
| | | $\frac{d}{dx}(14x^2-7xy+y^2)=28x-7x\frac{dy}{dx}-7y+2y\frac{dy}{dx}$ | A1 | | (= 0) |
| | | $2y\frac{dy}{dx}-7x\frac{dy}{dx}=7y-28x\rightarrow\frac{dy}{dx}=\frac{28x-7y}{7x-2y}$ www AG | A1 | **4** | As AG, intermed step nec |
| | (ii) | Subst $x$ = 1 into eqn curve & solve quadratic eqn in $y$ | M1 | | $(y=3$ or $4)$ |
| | | Subst $x$ = 1 and (one of) their $y$-value(s) into given $\frac{dy}{dx}$ | M1 | | $\left(\frac{dy}{dx}=7 \text{ or } 0\right)$ |
| | | Find eqn of tgt, with their $\frac{dy}{dx}$, going through (1, their $y$) | *M1 | | using (one of) $y$ value(s) |
| | | Produce either $y=7x-4$ or $y=4$ | A1 | | |
| | | Solve simultaneously their two equations | dep*M1 | | provided they have two |
| | | Produce $x=\frac{8}{7}$ | A1 | **6** | |
| | | | | **10** | |

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**9 (i)** $\frac{20}{k_1}$ (seconds) B1 **1**

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**(ii)** $\frac{d\theta}{dt} = -k_2(\theta - 20)$ B1 **1**

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**(iii)** Separate variables or invert each side M1 Correct eqn or very similar

Correct int of each side (+ $c$) A1,A1 for each integration

Subst $\theta = 60$ when $t = 0$ into eqn containing '$c$' M1 or $\theta = 60$ when $t =$ their (i)

$c$ (or $-c$) $= \ln 40$ or $\frac{1}{k_2}\ln 40$ or $\frac{1}{k_2}\ln 40k_2$ A1 Check carefully their '$c$'

Subst their value of $c$ and $\theta = 40$ back into equation M1 Use scheme on LHS

$t = \frac{1}{k_2}\ln 2$ A1 Ignore scheme on LHS

Total time = $\frac{1}{k_2}\ln 2 +$ their (i) (seconds) √A1 **8**

**SR** If the negative sign is omitted in part **(ii)**, allow all marks in **(iii)** with ln 2 replaced by $\ln \frac{1}{2}$.

**SR** If definite integrals used, allow M1 for eqn where $t = 0$ and $\theta = 60$ correspond; a second M1 for eqn where $t = t$ and $\theta = 40$ correspond & M1 for correct use of limits. Final answer scores 2.

**10**

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# 4725 Further Pure Mathematics 1

| | | | | |
|---|---|---|---|---|
| 1. | $984390625 - 25502500 = 958888125$ | B1<br>M1<br>A1 | <br><br>3<br>**3** | State correct value of $S_{250}$ or $S_{100}$<br>Subtract $S_{250} - S_{100}$ ( or $S_{101}$ or $S_{99}$ )<br>Obtain correct exact answer |
| 2. | $3a + 5b = 1,\ a + 2b = 1$<br><br>$a = -3,\ b = 2$ | M1<br>M1<br>A1 A1 | <br><br>4<br>**4** | Obtain a pair of simultaneous equations<br>Attempt to solve<br>Obtain correct answers. |
| 3. | (i) $11 - 29i$<br><br>(ii) $1 + 41i$ | B1 B1<br><br>B1 B1 | 2<br><br>2<br>**4** | Correct real and imaginary parts<br><br>Correct real and imaginary parts |
| 4. | Either $p + q = -1, pq = -8$<br><br>$\frac{p+q}{pq}$<br><br>$-\frac{7}{8}$<br><br>Or $\frac{1}{p} + \frac{1}{q} = 8$<br><br>$p + q = 1$<br><br>$-\frac{7}{8}$<br><br>Or $\frac{-1 \pm \sqrt{33}}{2}$<br><br>$-\frac{7}{8}$ | B1<br><br>B1<br><br>M1<br>A1<br><br>B1<br><br>B1<br><br>M1<br>A1<br><br>M1<br><br>A1<br>M1<br>A1 | <br><br><br><br>4<br>**4**<br><br><br><br><br><br><br><br><br><br><br><br> | Both values stated or used<br><br>Correct expression seen<br><br>Use their values in their expression<br>Obtain correct answer<br><br>Substitute $x = \frac{1}{u}$ and use new quadratic<br>Correct value stated<br><br>Use their values in given expression<br>Obtain correct answer<br><br>Find roots of given quadratic equation<br>Correct values seen<br>Use their values in given expression<br>Obtain correct answer |
| 5. | (i) $u^3 = \{(-)(5u + 7)\}^2$<br><br>$u^3 - 25u^2 - 70u - 49 = 0$<br><br>(ii)<br><br>-70 | M1<br>A1<br><br>A1<br><br>M1<br><br><br><br>A1 ft | <br><br><br>3<br><br><br><br>2<br>**5** | Use given substitution and rearrange<br>Obtain correct expression, or equivalent<br><br>Obtain correct final answer<br><br>Use coefficient of $u$ of their cubic or identity connecting the symmetric functions and substitute values from given equation<br><br>Obtain correct answer |

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|---|---|---|---|---|
| 6. | (i) $3\sqrt{2}, -\frac{\pi}{4}$ or $-45°$ AEF | B1 B1 | 2 | State correct answers |
| | (ii)(a) | B1B1<br>B1 ft | 3 | Circle, centre (3, -3),<br>through $O$ ft for $(\pm3,\pm3)$ only |
| | (ii)(b) | B1<br>B1<br>B1 | 3 | Straight line with +ve slope,<br>through (3, -3) or their centre<br>Half line only starting at centre |
| | (iii) | B1ft<br>B1ft<br>B1ft | 3<br>**11** | Area above horizontal through $a$,<br>below (ii) (b)<br>Outside circle |
| 7. | (i) | M1<br>A1 | 2 | Show that terms cancel in pairs<br>Obtain given answer correctly |
| | (ii) | M1<br>A1 | 2 | Attempt to expand and simplify<br>Obtain given answer correctly |
| | (iii) | B1 B1 | | Correct $\sum r$ stated $\sum 1 = n$ |
| | | M1* | | Consider sum of 4 separate terms on RHS |
| | | *DM1 | | Required sum is LHS – 3 terms |
| | $(n+1)^4 - 1 - n(n+1)(2n+1) - 2n(n+1) - n$ | A1 | | Correct unsimplified expression |
| | $4\sum_{r=1}^{n} r^3 = n^2(n+1)^2$ | A1 | 6<br>**10** | Obtain given answer correctly |
| 8. | (i) | B1<br>B1<br>B1 | 3 | Find coordinates (0, 0) (3, 1) (2, 1)<br>(5, 2 ) found<br>Accurate diagram sketched |
| | (ii) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ | B1 B1 | 2 | Each column correct |
| | (iii) *Either* | B1 | | Correct inverse for their (ii) stated |
| | $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ | M1 | | Post multiply **C** by inverse of (ii) |
| | | A1ft | | Correct answer found |
| | Or | M1 | | Set up 4 equations for elements from correct matrix multiplication |
| | | A2ft | | All elements correct, -1 each error |
| | | B1<br>B1<br>B1 | 6<br>**11** | Shear,<br>$x$ axis invariant or parallel to $x$-axis<br>eg image of (1, 1) is (3, 1)<br>**SR** allow s.f. 2 or shearing angle of correct angle to appropriate axis |

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|---|---|---|---|---|
| 9. | (i) $a\begin{vmatrix} a & 1 \\ 1 & 2 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & a \\ 1 & 1 \end{vmatrix}$<br>$2a^2 - 2a$ | M1<br>A1<br><br>A1 | <br><br>3 | Correct expansion process shown<br>Obtain correct unsimplified expression<br><br>Obtain correct answer |
| | (ii)<br>$a = 0$ or $1$ | M1<br>A1ft<br>A1ft | <br><br>3 | Equate their det to 0<br>Obtain correct answers, ft solving a quadratic |
| | (iii) (a) | B1 B1 | | Equations consistent, but non unique solutions |
| | (b) | B1<br>B1 | <br>4<br>**10** | Correct equations seen &<br>inconsistent, no solutions |
| 10. | i)<br>$u_2 = 7$ $u_3 = 19$ | M1<br>A1<br>A1 | <br><br>3 | Attempt to find next 2 terms<br>Obtain correct answers<br>Show given result correctly |
| | (ii)<br>$u_n = 2(3^{n-1}) + 1$ | M1<br>A1 | <br>2 | Expression involving a power of 3<br>Obtain correct answer |
| | (iii)<br>$u_{n+1} = 3(2(3^{n-1})+1) - 2$<br>$u_{n+1} = 2(3^n) + 1$ | B1ft<br>M1<br><br>A1<br>A1<br>B1 | <br><br><br><br><br>5<br>**10** | Verify result true when $n = 1$ or $n = 2$<br>Expression for $u_{n+1}$ using recurrence relation<br>Correct unsimplified answer<br>Correct answer in correct form<br>Statement of induction conclusion |

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# 4726 Further Pure Mathematics 2

| | | | |
|---|---|---|---|
| **1(i)** | Attempt area = ±Σ(0.3$y$) for at least three $y$ values | M1 | May be implied |
| | Get 1.313(1..) or 1.314 | A1 | Or greater accuracy |
| **(ii)** | Attempt ± sum of areas (4 or 5 values) | M1 | May be implied |
| | Get 0.518(4..) | A1 | Or greater accuracy |
| | | | **SC** |
| | | | If answers only seen, |
| | | | 1.313(1..) or 1.314 B2 |
| | | | 0.518(4..) B2 |
| | | | −1.313(1..) or −1.314 B1 |
| | | | −0.518(4..) B1 |
| | Or | | |
| | Attempt answer to part **(i)**−final rectangle | M1 | |
| | Get 0.518(4..) | A1 | |
| **(iii)** | Decrease width of strips | B1 | Use more strips or equivalent |
| **2** | Attempt to set up quadratic in $x$ | M1 | Must be quadratic; = 0 may be implied |
| | Get $x^2(y\text{-}1) - x(2y+1) + (y\text{-}1)=0$ | A1 | |
| | Use $b^2 \geq 4ac$ for real $x$ on their quadratic | M1 | Allow =,>,<,≤ here; may be implied |
| | Clearly solve to AG | A1 | If other (in)equalities used, the step to AG must be clear |
| | | | **SC** |
| | | | Reasonable attempt to diff. using prod/quot rule M1 |
| | | | Solve correct $dy/dx=0$ to get $x=-1$, $y=¼$ A1 |
| | | | Attempt to justify inequality e.g. graph or to show $d^2y/dx^2>0$ M1 |
| | | | Clearly solve to AG A1 |
| **3(i)** | Reasonable attempt at chain rule | M1 | Product in answer |
| | Reasonable attempt at product/quotient rule | M1 | Sum of two parts |
| | Correctly get $f'(0)=1$ | A1 | |
| | Correctly get $f''(0)=1$ | A1 | **SC** |
| | | | Use of $\ln y = \sin x$ follows same scheme |
| **(ii)** | Reasonable attempt at Maclaurin with their values | M1 | In $af(0) + bf'(0)x + cf''(0)x^2$ |
| | Get $1 + x + ½x^2$ | A1√ | From their f(0), f′(0), f″(0) in a correct Maclaurin; all non-zero terms |
| **4** | Attempt to divide out. | M1 | Or $A+B/(x-2)+(Cx(+D))/(x^2+4)$; allow $A=1$ and/or $B=1$ quoted |
| | Get $x^3=$ $A(x\text{-}2)(x^2+4)+B(x^2+4)+(Cx+D)(x\text{-}2)$ | M1 | Allow √ mark from their Part Fract; allow $D=0$ but not $C=0$ |
| | State/derive/quote $A=1$ | A1 | |
| | Use $x$ values and/or equate coeff | M1 | To potentially get all their constants |

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|---|---|---|---|
| | Get $B$=1, $C$=1, $D$=-2 | A1<br>A1 | For one other correct from cwo<br>For all correct from cwo |
| **5(i)** | Derive/quote $d\theta=2dt/(1+t^2)$ | B1 | May be implied |
| | Replace their $\cos\theta$ and their $d\theta$, both in terms of $t$ | M1 | Not $d\theta = dt$ |
| | Clearly get $\int(1-t^2)/(1+t^2)\,dt$ or equiv | A1 | Accept limits of $t$ quoted here |
| | Attempt to divide out | M1 | Or use AG to get answer above |
| | Clearly get/derive AG | A1 | |
| | **SC**<br>Derive $d\theta = 2\cos^2\frac{1}{2}\theta\,dt$ | B1 | |
| | Replace $\cos\theta$ in terms of half-angles and their $d\theta$ ($\neq dt$) | M1 | |
| | Get $\int 2\cos^2\frac{1}{2}\theta - 1\,dt$ or $\int 1 - 1/2\cos^2\frac{1}{2}\theta\,.2/(1+t^2)\,dt$ | A1 | |
| | Use $\sec^2\frac{1}{2}\theta = 1+t^2$ | M1 | |
| | Clearly get/derive AG | A1 | |
| **(ii)** | Integrate to $a\tan^{-1}bt - t$ | M1 | |
| | Get $\frac{1}{2}\pi - 1$ | A1 | |
| **6** | Get $k\sinh^{-1}k_1 x$ | M1 | For either integral; allow attempt at ln version here |
| | Get $\frac{1}{3}\sinh^{-1}\frac{3}{4}x$ | A1 | Or ln version |
| | Get $\frac{1}{2}\sinh^{-1}\frac{2}{3}x$ | A1 | Or ln version |
| | Use limits in their answers | M1 | |
| | Attempt to use correct ln laws to set up a solvable equation in $a$ | M1 | |
| | Get $a = 2^{\frac{1}{3}}.3^{\frac{1}{2}}$ | A1 | Or equivalent |

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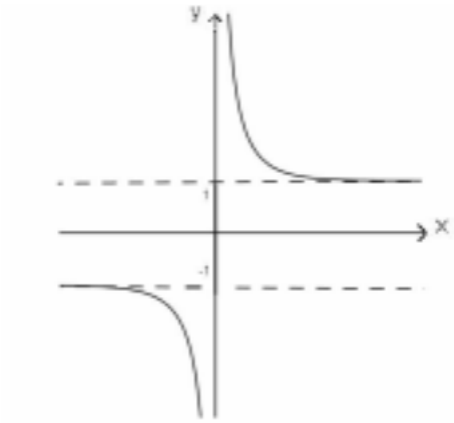
| **7(i)** |  | B1 | *y*-axis asymptote; equation may be implied if clear |
| | | B1 | Shape |
| | | B1 | $y = \pm 1$ asymptotes; may be implied if seen as on graph |
| **(ii)** | Reasonable attempt at product rule, giving two terms | M1 | |
| | Use correct Newton-Raphson at least once with their f′(*x*) to produce an $x_2$ | M1 | May be implied |
| | Get $x_2 = 2.0651$ | A1√ | One correct at any stage if reasonable |
| | Get $x_3 = 2.0653$, $x_4 = 2.0653$ | A1 | cao; or greater accuracy which rounds |
| **(iii)** | Clearly derive coth $x = \frac{1}{2}x$ | B1 | AG; allow derivation from AG<br>Two roots only |
| | Attempt to find second root e.g. symmetry | M1 | |
| | Get ± 2.0653 | A1√ | ± their iteration in part **(ii)** |
| **8(i)** | **(a)** Get $\frac{1}{2}(e^{\ln a} + e^{-\ln a})$ | M1 | |
| | Use $e^{\ln a} = a$ and $e^{-\ln a} = 1/a$ | M1 | |
| | Clearly derive AG | A1 | |
| | **(b)** Reasonable attempt to multiply out their attempts at exponential definitions of cosh and sinh | M1 | 4 terms in each |
| | Correct expansion seen as $e^{(x+y)}$ etc. | A1 | |
| | Clearly tidy to AG | A1 | With $e^{-(x-y)}$ seen or implied |
| **(ii)** | Use $x = y$ and cosh0 = 1 to get AG | B1 | |
| **(iii)** | Attempt to expand and equate coefficients | M1 | $(13 = R\cosh\ln a = R(a^2+1)/2a$<br>$5 = R\sinh\ln a = R(a^2-1)/2a)$ |
| | Attempt to eliminate *R* (or *a*) to set up a solvable equation in *a* (or *R*) | M1 | **SC**<br>If exponential definitions used, $8e^x + 18e^{-x} = Re^x/a + Rae^{-x}$ and same scheme follows |
| | Get $a = 3/2$ (or $R = 12$) | A1 | |
| | Replace for *a* (or *R*) in relevant equation to set up solvable equation in *R* (or *a*) | M1 | |
| | Get $R = 12$ (or $a = 3/2$) | A1 | Ignore if $a = 2/3$ also given |
| **(iv)** | Quote/derive $(\ln 3/2, 12)$ | B1√<br>B1√ | On their *R* and *a* |
| **9(i)** | Use $\sin\theta.\sin^{n-1}\theta$ and parts | M1 | Reasonable attempt with 2 parts, one yet to be integrated |

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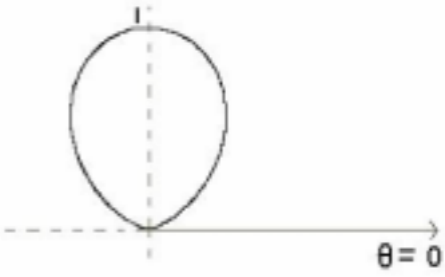
|---|---|---|---|
| | Get $-\cos\theta.\sin^{n-1}\theta+(n-1)\int\sin^{n-2}\theta.\cos^2\theta\ d\theta$ | A1 | Signs need to be carefully considered |
| | Replace $\cos^2 = 1 - \sin^2$ | M1 | |
| | Clearly use limits and get AG | A1 | |
| **(ii)** | **(a)** Solve for $r$=0 for at least one $\theta$ | M1 | $\theta$ need not be correct |
| | Get ($\theta$) = 0 and π | A1 | Ignore extra answers out of range |
| |   | B1 | General shape (symmetry stated or approximately seen) |
| | | B1 | Tangents at $\theta$=0, π and max $r$ seen |
| | **(b)**Correct formula used; correct $r$ | M1 | May be $\int r^2\ d\theta$ with correct limits |
| | Use $6I_6 = 5I_4$, $4I_4 = 3I_2$ | M1 | At least one |
| | Attempt $I_0$ (or $I_2$) | M1 | ($I_0 = \frac{1}{2}\pi$) |
| | Replace their values to get $I_6$ | M1 | |
| | Get 5π/32 | A1 | |
| | Use symmetry to get 5π/32 | A1 | May be implied but correct use of limits must be given somewhere in answer |
| | Or | | |
| | Correct formula used; correct $r$ | M1 | |
| | Reasonable attempt at formula $(2i\sin\theta)^6 = (z - {}^1/z)^6$ | M1 | |
| | Attempt to multiply out both sides (7 terms) | M1 | |
| | Get correct expansion | A1 | |
| | Convert to trig. equivalent and integrate their expression | M1 | cwo |
| | Get 5π/32 | A1 | |
| | Or | | |
| | Correct formula used; correct $r$ | M1 | |
| | Use double-angle formula and attempt to cube (4 terms) | M1 | |
| | Get correct expression | A1 | |
| | Reasonable attempt to put $\cos^2 2\theta$ into integrable form and integrate | M1 | |
| | Reasonable attempt to integrate $\cos^3 2\theta$ as e.g.$\cos^2 2\theta.\cos 2\theta$ | M1 | cwo |
| | Get 5π/32 | A1 | |

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# 4727 Further Pure Mathematics 3

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|---|---|---|---|---|
| 1 | $\left(\frac{1}{2}\sqrt{3}+\frac{1}{2}\mathrm{i}\right)^{\frac{1}{3}}=\left(\cos\frac{1}{6}\pi+\mathrm{i}\sin\frac{1}{6}\pi\right)^{\frac{1}{3}}$ | B1 | | For $\arg z=\frac{1}{6}\pi$ seen or implied |
| | $=\cos\frac{1}{18}\pi+\mathrm{i}\sin\frac{1}{18}\pi$, | M1 | | For dividing $\arg z$ by 3 |
| | $\cos\frac{13}{18}\pi+\mathrm{i}\sin\frac{13}{18}\pi$, | A1 | | For any one correct root |
| | $\cos\frac{25}{18}\pi+\mathrm{i}\sin\frac{25}{18}\pi$ | A1 | 4 | For 2 other roots and no more in range 0 ,, $\theta<2\pi$ |
| | | $\boxed{4}$ | | |
| 2 (i) | $\frac{1}{5}\mathrm{e}^{-\frac{1}{3}\pi\mathrm{i}}$ | B1 | 1 | For stating correct inverse in the form $r\,\mathrm{e}^{\mathrm{i}\theta}$ |
| (ii) | $r_1\mathrm{e}^{\mathrm{i}\theta}\times r_2\mathrm{e}^{\mathrm{i}\phi}=r_1r_2\,\mathrm{e}^{\mathrm{i}(\theta+\phi)}$ | M1<br>A1 | <br>2 | For stating 2 distinct elements multiplied<br>For showing product of correct form |
| (iii) | $Z^2=\mathrm{e}^{2\mathrm{i}\gamma}$ | B1 | | For $\mathrm{e}^{2\mathrm{i}\gamma}$ seen or implied |
| | $\Rightarrow\ \mathrm{e}^{2\mathrm{i}\gamma-2\pi\mathrm{i}}$ | B1 | 2 | For correct answer. aef |
| | | $\boxed{5}$ | | |
| 3 (i) | $[6-4\lambda,-7+8\lambda,-10+7\lambda]$ on $p$ | B1 | | For point on $l$ seen or implied |
| | $\Rightarrow 3(6-4\lambda)-4(-7+8\lambda)-2(-10+7\lambda)=8$ | M1 | | For substituting into equation of $p$ |
| | $\Rightarrow\ \lambda=1\ \Rightarrow\ (2,1,-3)$ | A1 | 3 | For correct point. Allow position vector |
| (ii) | METHOD 1 | | | |
| | $\mathbf{n}=[-4,8,7]\times[3,-4,-2]$ | M1*<br>M1<br>(*dep) | | For direction of $l$ and normal of $p$ seen<br>For attempting to find $\mathbf{n}_1\times\mathbf{n}_2$ |
| | $\mathbf{n}=k[12,13,-8]$ | A1 | | For correct vector |
| | $(2,1,-3)$ OR $(6,-7,-10)$ | M1 | | For finding scalar product of their point on $l$ with their attempt at $\mathbf{n}$, or equivalent |
| | $\Rightarrow\ 12x+13y-8z=61$ | A1 | 5 | For correct equation, aef cartesian |
| | METHOD 2 | | | |
| | $\mathbf{r}=[2,1,-3]$ *OR* $[6,-7,-10]$<br>$+\lambda[-4,8,7]+\mu[3,-4,-2]$ | M1<br>A1√ | | For stating eqtn of plane in parametric form (may be implied by next stage), using $[2,1,-3]$ (ft from (i)) Or $[6,-7,-10]$, $\mathbf{n}_1$ and $\mathbf{n}_2$ (as above) |
| | $x=\ 2-4\lambda+3\mu$<br>$y=\ 1+8\lambda-4\mu$<br>$z=-3+7\lambda-2\mu$ | M1<br>M1 | | For writing as 3 linear equations<br>For attempting to eliminate $\lambda$ and $\mu$ |
| | $\Rightarrow\ 12x+13y-8z=61$ | A1 | | For correct equation aef cartesian |
| | METHOD 3 | | | |
| | $3(6+3\mu)-4(-7-4\mu)-2(-10-2\mu)=8$ | M1 | | For finding foot of perpendicular from point on $l$ to $p$ |
| | $\Rightarrow\ \mu=-2\ \Rightarrow\ (0,1,-6)$ | A1 | | For correct point or position vector |
| | From 3 points $(2,1,-3)$, $(6,-7,-10)$, $(0,1,-6)$,<br>$\mathbf{n}$ = vector product of 2 of<br>$[2,0,3]$, $[6,-8,-4]$, $[-4,8,7]$ | M1 | | Use vector product of 2 vectors in plane |
| | $\Rightarrow\ \mathbf{n}=k[12,13,-8]$ | | | |
| | $(2,1,-3)$ OR $(6,-7,-10)$ | M1 | | For finding scalar product of their point on $l$ with their attempt at $\mathbf{n}$, or equivalent |
| | $\Rightarrow\ 12x+13y-8z=61$ | A1 | | For correct equation aef cartesian |
| | | $\boxed{8}$ | | |

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| 4 | (i) | IF $e^{\int \frac{1}{1-x^2}\,dx} = e^{\frac{1}{2}\ln\frac{1+x}{1-x}} = \left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$ | M1<br>A1 | 2 | For IF stated or implied. Allow $\pm\int$ and omission of $dx$<br>For integration and simplification to **AG**<br>(intermediate step must be seen) |
| | (ii) | $\frac{d}{dx}\left(y\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}\right) = (1+x)^{\frac{1}{2}}$ | M1* | | For multiplying both sides by IF |
| | | $y\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}} = \frac{2}{3}(1+x)^{\frac{3}{2}} + c$ | M1<br>A1 | | For integrating RHS to $k(1+x)^n$<br>For correct equation (including + $c$) |
| | | $(0, 2) \Rightarrow 2 = \frac{2}{3} + c \Rightarrow c = \frac{4}{3}$ | M1 (*dep)<br>M1 (*dep) | | In either order:<br>For substituting (0, 2) into their GS (including +$c$)<br>For dividing solution through by IF, including dividing $c$ or their numerical value for $c$ |
| | | $y = \frac{2}{3}(1+x)(1-x)^{\frac{1}{2}} + \frac{4}{3}\left(\frac{1-x}{1+x}\right)^{\frac{1}{2}}$ | A1 | 6 | For correct solution<br>aef (even unsimplified) in form $y = f(x)$ |
| | | | | **8** | |
| 5 | (i) | $m^2 - 6m + 9\ (=0) \Rightarrow m = 3$ | M1<br>A1 | | For attempting to solve correct auxiliary equation<br>For correct $m$ |
| | | CF = $(A + Bx)e^{3x}$ | A1 | 3 | For correct CF |
| | (ii) | $ke^{3x}$ and $kxe^{3x}$ both appear in CF | B1 | 1 | For correct statement |
| | (iii) | $y = kx^2e^{3x} \Rightarrow y' = 2kxe^{3x} + 3kx^2e^{3x}$ | M1<br>A1 | | For differentiating $kx^2e^{3x}$ twice<br>For correct $y'$ aef |
| | | $\Rightarrow y'' = 2ke^{3x} + 12kxe^{3x} + 9kx^2e^{3x}$ | A1 | | For correct $y''$ aef |
| | | $\Rightarrow$<br>$ke^{3x}(2 + 12x + 9x^2 - 12x - 18x^2 + 9x^2) = e^{3x}$ | M1 | | For substituting $y''$, $y'$, $y$ into DE |
| | | $\Rightarrow k = \frac{1}{2}$ | A1 | 5 | For correct $k$ |
| | | | | **9** | |

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| 6 (i) | METHOD 1 | | | |
| | $\mathbf{n}_1 = [1,1,0]\times[1,-5,-2]$ | M1 | | For attempting to find vector product of the pair of direction vectors |
| | $= [-2,2,-6] = k[1,-1,3]$ | A1 | | For correct $\mathbf{n}_1$ |
| | Use $(2,2,1)$ | M1 | | For substituting a point into equation |
| | $\Rightarrow \mathbf{r}.[-2,2,-6] = -6 \Rightarrow \mathbf{r}.[1,-1,3] = 3$ | A1 | 4 | For correct equation. aef in this form |
| | METHOD 2 | | | |
| | $x = 2+\lambda+\mu$<br>$y = 2+\lambda-5\mu$<br>$z = 1 \quad -2\mu$ | M1 | | For writing as 3 linear equations |
| | | M1 | | For attempting to eliminate $\lambda$ and $\mu$ |
| | $\Rightarrow x-y+3z = 3$ | A1 | | For correct cartesian equation |
| | $\Rightarrow \mathbf{r}.[1,-1,3] = 3$ | A1 | | For correct equation. aef in this form |
| (ii) | For $\mathbf{r} = \mathbf{a}+t\mathbf{b}$ | | | |
| | METHOD 1 | | | |
| | $\mathbf{b} = [1,-1,3]\times[7,17,-3]$ | M1 | | For attempting to find $\mathbf{n}_1 \times \mathbf{n}_2$ |
| | $= k[2,-1,-1]$ | A1√ | | For a correct vector. ft from $\mathbf{n}_1$ in **(i)** |
| | e.g. $x$, $y$ or $z = 0$ in $\begin{cases} x- \quad y+3z = 3 \\ 7x+17y-3z = 21 \end{cases}$ | M1 | | For attempting to find a point on the line |
| | $\Rightarrow \mathbf{a} = \left[0, \frac{3}{2}, \frac{3}{2}\right]$ OR $[3,0,0]$ OR $[1,1,1]$ | A1√ | | For a correct vector. ft from equation in **(i)**<br>**SR** a correct vector may be stated without working |
| | Line is (e.g.) $\mathbf{r} = [1,1,1]+t[2,-1,-1]$ | A1√ | 5 | For stating equation of line ft from **a** and **b**<br>**SR** for **a** $= [2,2,1]$ stated award M0 |
| | METHOD 2 | | | |
| | Solve $\begin{cases} x- \quad y+3z = 3 \\ 7x+17y-3z = 21 \end{cases}$<br>by eliminating one variable (e.g. $z$) | M1 | | In either order:<br>For attempting to solve equations |
| | Use parameter for another variable (e.g. $x$) to find other variables in terms of $t$ | M1 | | For attempting to find parametric solution |
| | (eg) $y = \frac{3}{2}-\frac{1}{2}t$, $z = \frac{3}{2}-\frac{1}{2}t$ | A1√ | | For correct expression for one variable |
| | | A1√ | | For correct expression for the other variable<br>ft from equation in **(i)** for both |
| | Line is (eg) $\mathbf{r} = \left[0, \frac{3}{2}, \frac{3}{2}\right]+t[2,-1,-1]$ | A1√ | | For stating equation of line. ft from parametric solutions |
| | METHOD 3 | | | |
| | eg $x$, $y$ or $z = 0$ in $\begin{cases} x- \quad y+3z = 3 \\ 7x+17y-3z = 21 \end{cases}$ | M1 | | For attempting to find a point on the line |
| | $\Rightarrow \mathbf{a} = \left[0, \frac{3}{2}, \frac{3}{2}\right]$ *OR* $[3,0,0]$ *OR* $[1,1,1]$ | A1√ | | For a correct vector. ft from equation in **(i)**<br>**SR** a correct vector may be stated without working<br>**SR** for **a** $= [2,2,1]$ stated award M0 |
| | eg $[3,0,0]-[1,1,1]$ | M1 | | For finding another point on the line and using it with the one already found to find **b** |
| | $\mathbf{b} = k[2,-1,-1]$ | A1√ | | For a correct vector. ft from equation in **(i)** |
| | Line is (eg) $\mathbf{r} = [1,1,1]+t[2,-1,-1]$ | A1√ | | For stating equation of line. ft from **a** and **b** |

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| **6 (ii) contd** | METHOD 4 | | | |
| | A point on $\Pi_1$ is $[2+\lambda+\mu, 2+\lambda-5\mu, 1-2\mu]$ | M1 | | For using parametric form for $\Pi_1$ and substituting into $\Pi_2$ |
| | On $\Pi_2 \Rightarrow$ $[2+\lambda+\mu, 2+\lambda-5\mu, 1-2\mu].[7, 17, -3] = 21$ | A1 | | For correct unsimplified equation |
| | $\Rightarrow \lambda - 3\mu = -1$ | A1 | | For correct equation |
| | Line is (e.g.) $\mathbf{r} = [2, 2, 1] + (3\mu - 1)[1, 1, 0] + \mu[1, -5, -2]$ | M1 | | For substituting into $\Pi_1$ for $\lambda$ *or* $\mu$ |
| | $\Rightarrow \mathbf{r} = [1, 1, 1]$ *or* $\left[\frac{7}{3}, \frac{1}{3}, \frac{1}{3}\right] + t[2, -1, -1]$ | A1 | | For stating equation of line |
| | | | **9** | |
| **7 (i)** | $\cos 3\theta + i\sin 3\theta = c^3 + 3ic^2s - 3cs^2 - is^3$ | M1 | | For using de Moivre with $n = 3$ |
| | $\Rightarrow \cos 3\theta = c^3 - 3cs^2$ and $\sin 3\theta = 3c^2s - s^3$ | A1 | | For both expressions in this form (seen or implied) **SR** For expressions found without de Moivre M0 A0 |
| | $\Rightarrow \tan 3\theta = \dfrac{3c^2s - s^3}{c^3 - 3cs^2}$ | M1 | | For expressing $\dfrac{\sin 3\theta}{\cos 3\theta}$ in terms of $c$ and $s$ |
| | $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} = \dfrac{\tan\theta(3 - \tan^2\theta)}{1 - 3\tan^2\theta}$ | A1 | **4** | For simplifying to **AG** |
| **(ii) (a)** | $\theta = \frac{1}{12}\pi \Rightarrow \tan 3\theta = 1$ $\Rightarrow 1 - 3t^2 = t(3 - t^2) \Rightarrow$ $t^3 - 3t^2 - 3t + 1 = 0$ | B1 | **1** | For both stages correct **AG** |
| **(b)** | $(t+1)(t^2 - 4t + 1) = 0$ | M1 | | For attempt to factorise cubic |
| | | A1 | | For correct factors |
| | $\Rightarrow (t = -1),\ t = 2 \pm \sqrt{3}$ | A1 | | For correct roots of quadratic |
| | $-$ sign for smaller root $\Rightarrow$ $\tan\frac{1}{12}\pi = 2 - \sqrt{3}$ | A1 | **4** | For choice of $-$ sign and correct root **AG** |
| **(iii)** | $dt = (1 + t^2)\, d\theta$ | B1 | | For differentiation of substitution and use of $\sec^2\theta = 1 + \tan^2\theta$ |
| | $\Rightarrow \int_0^{\frac{1}{12}\pi} \tan 3\theta\, d\theta$ | B1 | | For integral with correct $\theta$ limits seen |
| | $= \left[\frac{1}{3}\ln(\sec 3\theta)\right]_0^{\frac{1}{12}\pi} = \frac{1}{3}\ln\left(\sec\frac{1}{4}\pi\right)$ | M1 | | For integrating to $k\ln(\sec 3\theta)$ OR $k\ln(\cos 3\theta)$ |
| | $= \frac{1}{3}\ln\sqrt{2} = \frac{1}{6}\ln 2$ | M1 | | For substituting limits and $\sec\frac{1}{4}\pi = \sqrt{2}$ OR $\cos\frac{1}{4}\pi = \frac{1}{\sqrt{2}}$ seen |
| | | A1 | **5** | For correct answer aef |
| | | | **14** | |

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| 8 | (i) | $a^2 = (ap)^2 = apap \Rightarrow a = pap$ | B1 | | For use of given properties to obtain **AG** |
| | | $p^2 = (ap)^2 = apap \Rightarrow p = apa$ | B1 | 2 | For use of given properties to obtain **AG**<br>**SR** allow working from **AG** to obtain relevant properties |
| | (ii) | $(p^2)^2 = p^4 = e \Rightarrow$ order $p^2 = 2$ | B1 | | For correct order with no incorrect working seen |
| | | $(a^2)^2 = (p^2)^2 = e \Rightarrow$ order $a = 4$ | B1 | | For correct order with no incorrect working seen |
| | | $(ap)^4 = a^4 = e \Rightarrow$ order $ap = 4$ | B1 | | For correct order with no incorrect working seen |
| | | $(ap^2)^2 = ap^2ap^2 = ap\,.\,a\,.\,p = a^2$ | M1 | | For relevant use of **(i)** or given properties |
| | | *OR* $ap^2 = a.a^2 = a^3 \Rightarrow$<br>$(ap^2)^2 = a^6 = a^2$ | A1 | 5 | For correct order with no incorrect working seen |
| | | $\Rightarrow$ order $ap^2 = 4$ | | | |
| | (iii) | METHOD 1<br>$p^2 = a^2,\ ap^2 = a^3$ | M2 | | For use of the given properties to simplify $p^2$ and $ap^2$ |
| | | $\Rightarrow \{e, a, p^2, ap^2\} = \{e, a, a^2, a^3\}$ | A1 | | For obtaining $a^2$ and $a^3$ |
| | | which is a cyclic group | A1 | 4 | For justifying that the set is a group |
| | | METHOD 2 | | | |
| | | $\begin{array}{c\|cccc} & e & a & p^2 & ap^2 \\ \hline e & e & a & p^2 & ap^2 \\ a & a & p^2 & ap^2 & e \\ p^2 & p^2 & ap^2 & e & a \\ ap^2 & ap^2 & e & a & p^2 \end{array}$ | M1<br>A1 | | For attempting closure with all 9 non-trivial products seen<br>For all 16 products correct |
| | | Completed table is a cyclic group | B2 | | For justifying that the set is a group |
| | | METHOD 3 | | | |
| | | $\begin{array}{c\|cccc} & e & a & p^2 & ap^2 \\ \hline e & e & a & p^2 & ap^2 \\ a & a & p^2 & ap^2 & e \\ p^2 & p^2 & ap^2 & e & a \\ ap^2 & ap^2 & e & a & p^2 \end{array}$ | M1<br>A1 | | For attempting closure with all 9 non-trivial products seen<br>For all 16 products correct |
| | | Identity = $e$ | B1 | | For stating identity |
| | | Inverses exist since<br>EITHER: $e$ is in each row/column<br>OR: $p^2$ is self-inverse; $a, ap^2$ form an inverse pair | B1 | | For justifying inverses ($e^{-1} = e$ may be assumed) |

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| **(iv)** | METHOD 1<br>e.g. $\left.\begin{matrix} a\,.\,ap = a^2\,p = p^3 \\ ap\,.\,a = p \end{matrix}\right\} \Rightarrow$ not commutative | M1<br><br><br>M1<br>B1<br>A1 | **4** | For attempting to find a non-commutative pair of elements, at least one involving $a$ (may be embedded in a full or partial table)<br>For simplifying elements both ways round<br>For a correct pair of non-commutative elements<br>For stating $Q$ non-commutative, with a clear argument |
| | METHOD 2<br>Assume commutativity, so (eg) $ap = pa$<br>**(i)** $\Rightarrow$<br>$p = ap\,.a \Rightarrow p = pa\,.a = pa^2 = pp^2 = p^3$<br>But $p$ and $p^3$ are distinct<br>$\Rightarrow Q$ is non-commutative | M1<br><br>M1<br><br>B1<br>A1 | | For setting up proof by contradiction<br><br>For using **(i)** and/or given properties<br><br>For obtaining and stating a contradiction<br>For stating $Q$ non-commutative, with a clear argument |
| | | **15** | | |

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# 4728 Mechanics 1

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| 1 i | $x^2 + (3x)^2 = 6^2$<br>$10x^2 = 36$<br>$x = 1.9(0)$ (1.8973..) | M1<br>A1<br>A1<br>[3] | Using Pythagoras, 2 squared terms<br>May be implied<br>Not surd form unless rationalised $(3\sqrt{10})/5$, $(6\sqrt{10})/10$ |
| ii | $\tan\theta = 3x/x$ (= 3×1.9/1.9) = 3<br><br>$\theta = 71.6^o$ (71.565..) | M1<br><br>A2<br><br>[3] | Must target correct angle.<br>Accept sin $\theta$ = 3×1.9/6 or cos$\theta$ = 1.9/6 which give $\theta$=71.8°, θ=71.5° respectively, A1.<br>**SR** $\theta = 71.6^o$ from $\tan\theta = 3x/x$ if $x$ is incorrect; $x$ used A1, no evidence of $x$ used A2 |
| 2 i | | B1<br>B1<br>[2] | Inverted V shape with straight lines.<br>Starts at origin, ends on $t$-axis, or horizontal axis if no labelling evident |
| ii | <br>$6 = 3v/2$<br>$v = 4$ ms$^{-1}$ | M1<br>A1<br>A1<br>[3] | Not awarded if special (right angled, isosceles) triangle assumed, or<br>$s = (u+v)t/2$, or max $v$ at specific $t$. |
| iii | T accn = 4/2.4 or s accn = 16/(2x2.4)<br>T accn = 1 2/3 s or s accn = 10/3<br>Deceleration = 4/(3 - 1 2/3) or 16/2(6-10/3)<br>Deceleration = 3 ms$^{-2}$ | M1*<br>A1<br>D*M1<br>A1<br>[4] | Uses $t = v/a$ or $s = v^2/2a$.<br>May be implied<br>Accept 4/(3 - 1.67) or 16/2(6-3.33)<br>Accept 3.01; award however $v$ = 4 obtained in (ii). $a$ = -3 gets A0. |
| 3 i | 0.8gsin30<br>0.8x0.2<br>$0.8\times9.8\sin30 - T = 0.8\text{x}0.2$<br>$T = 3.76$ N AG | B1<br>B1<br>M1<br>A1<br>[4] | Not for 3.92 stated without justification<br>Or 0.16<br>Uses N2L // to slope, 3 non-zero terms, inc $ma$<br>Not awarded if initial B1 withheld. |
| ii | $3.76 - F = 3\times0.2$<br>$F = 3.16$<br>$3.16 = \mu\text{x}3\times9.8$<br><br>$\mu = 0.107$ (0.10748) | M1<br>A1<br>A1<br>M1<br><br>A1<br>[5] | Uses N2L, B alone, 3 non-zero terms<br>Needs *correct value* of $T$.<br>May be implied.<br>Uses $F = \mu R$ (Accept with $R$ = 3, but not with $R$=0.8g(cos30), $F$=0.6, $F$=3.76, $F = f$(mass $P$))<br>Not 0.11, 0.108 (unless it comes from using g=9.81 consistently through question. |

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| 4 i | $v^2 = 7^2 - 2\times9.8\times2.1$<br>$v = 2.8$ ms$^{-1}$ | M1<br>A1<br>A1<br>[3] | Uses $v^2 = u^2 - 2gs$. Accept $7^2 = u^2 + 2gs$ |
| ii | $v = 0$<br>$0^2 = 7^2 - 2\times9.8s$<br>$s = 2.5$ m | B1<br>M1<br>A1<br>[3] | Velocity = 0 at greatest height<br>Uses $0 = u^2 - 2gs$. Accept $7^2 = 2\times9.8s$. |
| iii | $v$ = -5.7 (or $t$ = 0.71 oef to reach greatest height)<br>$-5.7 = 7 - 9.8t$ or $5.7 = (0+) 9.8T$<br>$t$ = 1.3(0) s (1.2959..) | B1<br>M1<br>A1<br>[3] | Allows for change of direction<br>Uses $v = u +$ or $- gt$.<br>Not 1.29 unless obtained from g=9.81 consistently |
| 5 i | $0.5\times6 = 0.5v + m(v+1)$<br>$3 = 0.5v + mv + m$<br>$v(m + 0.5) = -m + 3$ AG | M1<br>A1<br>A1<br>[3] | Uses CoLM. Includes g throughout MR-1 |
| ii | Momentum before = +/- $(4m - 0.5\times2)$<br>+/- $(4m - 0.5\times2) = mv + 0.5(v+1)$<br>$4m - 0.5\times2 = mv + 0.5(v+1)$<br>$v(m+0.5) = 4m - 1.5$ | B1<br>M1<br>A1<br>A1<br>[4] | Includes g throughout MR-1<br>Needs opposite directions in CoLM on "before" side only.<br>RHS in format am + b or b + am. Ignore values for a and b if quoted. |
| iii | $4m - 1.5 = - m + 3$<br>$5m = 4.5$<br>$m = 0.9$ kg AG<br>$0.9 + v(0.9+0.5) = 3$ or $4\times0.9 - 1.5 = v(0.9+0.5)$<br>$v = (3-0.9)/(0.9+0.5) = 2.1/1.4$<br>$v = 1.5$ ms$^{-1}$ | M1<br><br>A1<br>M1<br><br><br>A1<br>[4] | Attempts to obtain eqn in 1 variable from answers in (i) and (ii)<br>Ignore $m$ = -0.5 if seen<br>Substitutes for $m$=0.9 in any $m$, $v$ equation obtained earlier. |
| 6 ia<br><br><br>b | Perp = 10cos20 (= 9.3967 or 9.4)<br>// = 10sin20 (= 3.4202)<br><br>$\mu$ = 10sin20/10cos20 = tan20 (= 3.42/9.4)<br>$\mu$ = 0.364 (0.36397..) AG | B1<br>B1<br>[2]<br>M1<br>A1<br>[2] | Includes g, MR -1 in part (i). Accept –ve values.<br><br>Must use $\|F\| = \mu\|R\|$<br>Accept after inclusion of g twice |
| ii | *No misread, and resolving of 10 and T required*<br>$R$ = 10cos20 + $T$cos45<br><br>$F$ = $T$cos45 - 10sin 20 or $T$cos45 = $\mu R$ + 10sin20<br>$T$cos45 – 3.42 = 0.364(9.4 + $T$cos45)<br>$0.707T - 3.42 = 3.42 + 0.257T$<br>$0.45T = 6.84$<br>$T$ = 15.2 N (15.209..) | M1*<br>A1<br>M1*<br>A1<br>D*M1<br>A1<br><br>A1<br><br>[7] | 3 term equation perp plane, 2 unknowns<br>$9.4 + 0.707T$ (accept $9.4+.71T$)<br>3 term equation // plane, 2 unknowns<br>$0.707T - 3.42$ (accept $0.71T - 3.4$)<br>Substitutes for $F$ and $R$ in $F$=0.364$R$<br><br><br>*Award final A1 only for T = 149 N after using 10g for weight* |

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| 7 i | $a = \mathrm{d}v/\mathrm{d}t$<br>$a = 6 - 2t$ ms$^{-2}$ | M1<br>A1<br>[2] | Differentiation attempt. Answer 6-$t$ implies division by $t$ |
| ii | $s = \int v\mathrm{d}t$<br>$s = \int 6t - t^2 \ \mathrm{d}t$<br>$s = 3t^2 - t^3/3$ (+$c$)<br>$t = 0$, $v = 0$, $c = 0$<br>$t = 3$, $s = 3\text{x}3^2 - 3^3/3$<br>$s =$ 18 m AG | M1*<br><br>A1<br>B1<br>D*M1<br>A1<br>[5] | Integration attempt on $v$<br><br><br>Award if limits 0,3 used<br>Requires earlier integration<br>Does not require B1 to be earned. |
| iii | Distance remaining (= 100 -18) = 82<br>Total time = 3 + 82/9<br>$T = 12.1$ s (12 1/9) | B1<br>M1<br>A1<br>[3] | <br>Numerator not 100<br>Not 109/9 |
| iv | Distance before slows = 18 + (22 - 3)x9<br>Distance while decelerating = 200 - 189 = 11<br><br>$11 = 9t - 0.3t^2$ or $11 = (9+8.23)t/2$ or $8.23 = 9-0.6t$<br><br>$t = 1.28$ (1.2765.., accept 1.3)<br>$T = 23.3$ s (23.276..) | M1*<br>A1<br>D*M1<br>A1<br>D*M1<br>A1<br>A1<br>[7] | (=189 m) Two sub-regions considered<br>Accept 10.99. 10.9 penalise -1PA.<br>Uses $s = ut - 0.5\text{x}0.6t^2$, or $v^2 = u^2 - 2\text{x}0.6s$ with $s=(u+v)t/2$ or $v=u+at$<br>Finds $t$. (If QE, it must have 3 terms and smaller positive root chosen.) |

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# 4729 Mechanics 2

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| **1 (i)** | ½×75×12² or ½×75×3² (either KE)<br>75×9.8×40 (PE)<br>$R$×180 ( change in energy = 24337)<br>½×75×12²=½×75×3²+75×9.8×40–$R$×180<br>$R$ = 135 N | B1<br>B1<br>B1<br>M1<br>A1 **5** | M1 $12^2 = 3^2 + 2a \times 180$<br>A1 $a = 0.375$ (3/8)<br>M1 $75 \times 9.8 \times \sin\theta - R = 75a$<br>A1 $R = 135$<br>(max 4 for no energy) **5** |

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| **2 (i)** | $R = F = P/v = 44\,000/v = 1400$<br>$v$ = 31.4 m s$^{-1}$ | M1<br>A1 **2** | |
| **(ii)** | $44\,000/v = 1400 + 1100 \times 9.8 \times 0.05$<br>$v$ = 22.7 m s$^{-1}$ | M1<br>A1<br>A1 **3** | must have g |
| **(iii)** | 22 000/10 + 1100×9.8×0.05 – 1400<br>= 1100$a$<br>$a$ = 1.22 m s$^{-2}$ | M1<br>A1<br>A1 **3** | **8** |

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| **3 (i)** | $\cos\theta = 5/13$ or $\sin\theta = 12/13$ or $\theta = 67.4°$<br>$0.5 \times F\sin\theta = 70 \times 1.4 + 50 \times 2.8$<br>$F$ = 516 N | B1<br>M1<br>A1<br>A1 **4** | any one of these<br>moments about $A$ (ok without 70)<br>$0.5\sin\theta = 0.4615$<br>**SR** 1 for 303 (omission of beam) |
| **(ii)** | $F\sin\theta = 120 + Y$ (resolving vertically)<br>$Y$ = 356 ✓ their F × 12/13 – 120<br>$X = F\cos\theta$ (resolving horizontally)<br>$X$ = 198 ✓ their $F$ × 5/13<br>Force = √(356² + 198²)<br>407 or 408 N | M1<br>A1 ✓<br>M1<br>A1 ✓<br>M1<br>A1 **6** | M1/A1 for moments<br>(B) $Y$×2.8+1.4×70=2.3×516✓×12/13<br>(C) $0.5 \times Y = 0.9 \times 70 + 2.3 \times 50$<br>(D) $1.2X = 1.4 \times 70 + 2.8 \times 50$<br>**10** |

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| **4 (i)** | $T = 0.4 \times 0.6 \times 2^2$<br>$T$ = 0.96 N | M1<br>A1 **2** | |
| **(ii)** | $S - T$<br>$S - T = 0.1 \times 0.3 \times 2^2$<br>$S$ = 1.08 | B1<br>M1<br>A1<br>A1 **4** | may be implied |
| **(iii)** | $v = r\omega$<br>$v_P = 0.6$<br>$v_B = 1.2$<br>½×0.1×0.6² + ½×0.4×1.2²<br>0.306 | M1<br>A1<br>A1<br>M1<br>A1 **5** | (0.018 + 0.288) separate speeds<br>**11** |

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| **5 (i)** | $\bar{d} = (2\times 6\sin\pi/4)/3\pi/4$<br>$\bar{d} = 3.60$ | M1<br>A1 **2** | must be correct formula with rads<br>AG |
| **(ii)** | $\bar{d}\cos 45° =$ "2.55"<br>$5\bar{x} = 3\times 3 + 2\times$ "2.55"<br>$\bar{x} = 2.82$<br>$5\bar{y} = 3\times 6 + 2\times(12 +$ "2.55")<br>$\bar{y} = 9.42$ | B1<br>M1<br>A1<br>A1<br>M1<br>A1<br>A1<br>**7** | may be implied<br>moments must not have areas<br>2kg/3kg misread (swap) gives (2.73,11.13) $\theta = 21.7°$<br>(MR – 2) (max 7 for (ii) + (iii))<br>SR -1 for $\bar{x}$, $\bar{y}$ swap |
| **(iii)** | $\tan\theta = 2.82/8.58$<br>$\theta = 18.2°$ | M1<br>A1 **2** | M0 for their $\bar{x}$ / $\bar{y}$<br>their $\bar{x}/(18 - \bar{y})$ **11** |

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| **6 (i)** | $I = 0.9 = 6\times 0.2 - v\times 0.2$<br>$v = 1.5$ | M1<br>A1<br>A1 **3** | needs to be mass 0.2 |
| **(ii)** | $0.6 = (c - b)/6$<br>$6\times 0.2 = 0.2b + 0.1c$<br>$b = 2.8$<br>$0.4\times 5 + 0.2\times 1.5 = 0.4a + 0.2\times 6$ or<br>$I = 0.9 = -0.4a - -\ 0.4\times 5$<br>$a = 2.75$<br>$2.75 < 2.8$<br>no further collision | M1<br>A1<br>M1<br>A1<br>A1<br>M1<br>A1<br>A1<br>M1<br>A1 **10** | restitution (allow 1.5 for M1)<br>momentum (allow 1.5 for M1)<br>1st collision (needs their 1.5 for M1)<br>compare $v$'s of $A$ and $B$ (calculated) **13** |

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| **7(i)** | $9 = 17\cos 25°\times t$<br>$t = 0.584$ (or $9/17\cos 25°$)<br>$d = 17\sin 25°\times 0.584 + \frac{1}{2}\times 9.8\times 0.584^2$ (d = $ht$ lost (5.87)<br>$h = 2.13$ | M1<br>A1<br>M1<br>A1<br>A1 **5** | B1 $y = x\tan\theta - 4.9x^2/v^2\cos^2\theta$<br>M1/A1 $y = 9\tan(-25°) - 4.9\times 9^2/17^2\cos^2 25°$<br>A1 y = -5.87<br>2.13 |
| **(ii)** | $v_h = 17\cos 25°$ (15.4)<br>$v_v = 17\sin 25° + 9.8\times 0.584$ or<br>$v_v^2 = (17\sin 25°)^2 + 2\times 9.8\times 5.87$<br>$v_v = 12.9$<br>$\tan\theta = 12.9/15.4$<br>$\theta = 40.0°$ below horizontal | B1<br>M1<br>A1<br>M1<br>A1 **5** | M1/A1 $dy/dx =$<br>$\tan\theta - 9.8x/v^2\cos^2\theta$<br>A1 $dy/dx = -0.838$<br>M1 $\tan^{-1}(-.838)$<br>or 50.0° to vertical |
| **(iii)** | speed = $\sqrt{(12.9^2 + 15.4^2)}$<br>$\frac{1}{2}mv^2 = \frac{1}{2}m\times 20.1^2\times 0.7$<br>$v = 16.8$ m s$^{-1}$ | M1<br>A1<br>M1<br>A1 **4** | (20.1)<br>NB 0.3 instead of 0.7 gives 11.0 (M0) **14** |

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# 4730 Mechanics 3

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| **1 i** | Horiz. comp. of vel. after impact is $4\text{ms}^{-1}$<br>Vert. comp. of vel. after impact is<br>$\sqrt{5^2 - 4^2} = 3\text{ms}^{-1}$<br>Coefficient of restitution is 0.5 | B1<br><br>B1<br>B1<br>[3] | May be implied<br><br>AG<br>From e = 3/6 |
| **ii** | Direction is vertically upwards<br>Change of velocity is 3 – (-6)<br>Impulse has magnitude 2.7Ns | B1<br>M1<br>A1<br>[3] | <br><br>From $m(\Delta v) = 0.3\times 9$ |
| **2 i** | Horizontal component is 14N<br><br><br>$80\times 1.5 = 14\times 1.5 + 3Y$ or<br>$3(80 - Y) = 80\times 1.5 + 14\times 1.5$ or<br>$1.5(80 - Y) = 14\times 0.75 + 14\times 0.75 + 1.5Y$<br>Vertical component is 33N upwards | B1<br><br>M1<br><br><br>A1<br>A1<br>[4] | <br>For taking moments for $AB$ about $A$ or $B$ or the midpoint of $AB$<br><br><br><br>AG |
| **ii** | Horizontal component at $C$ is 14N<br>[Vertical component at $C$ is<br>$(\pm)\sqrt{50^2 - 14^2}$ ]<br>[$W = (\pm)48 - 33$]<br>Weight is 15N | B1<br>M1<br>DM1<br>A1<br>[4] | May be implied<br>for using $R^2 = H^2 + V^2$<br>For resolving forces at $C$ vertically |
| **3 i** | <br>$4\times 3\cos 60^\circ - 2\times 3\cos 60^\circ = 2b$<br>$b = 1.5$<br>**j** component of vel. of $B = (-)3\sin 60^\circ$<br>[$v^2 = b^2 + (-3\sin 60^\circ)^2$]<br><br>Speed ($3\text{ms}^{-1}$) is unchanged<br>[Angle with l.o.c. = $\tan^{-1}(3\sin 60^\circ/1.5)$]<br>Angle is $60^\circ$. | M1<br>A1<br>A1<br>B1ft<br>M1<br><br>A1ft<br>M1<br>A1ft<br>[8] | For using the p.c.mmtm parallel to l.o.c.<br><br><br>ft consistent sin/cos mix<br>For using $v^2 = b^2 + v_y^2$<br><br>AG ft - allow same answer following consistent sin/cos mix.<br>For using angle = $\tan^{-1}(\pm v_y/v_x)$<br>ft consistent sin/cos mix |
| **ii** | [$e(3\cos 60^\circ + 3\cos 60^\circ) = 1.5$]<br>Coefficient is 0.5 | M1<br>A1ft<br>[2] | For using NEL<br>ft - allow same answer following consistent sin/cos mix throughout. |

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| **4 i** | $F - 0.25v^2 = 120v(\mathrm{d}v/\mathrm{d}x)$<br>$F = 8000/v$<br><br>$[32000 - v^3 = 480v^2(\mathrm{d}v/\mathrm{d}x)]$<br>$\frac{480v^2}{v^3 - 32000}\frac{\mathrm{d}v}{\mathrm{d}x} = -1$ | M1<br>A1<br>B1<br><br>M1<br><br>A1<br>[5] | For using Newton's second law with $a = v(\mathrm{d}v/\mathrm{d}x)$<br><br>For substituting for $F$ and multiplying throughout by $4v$ (or equivalent)<br><br>AG |
| **ii** | $\int \frac{480v^2}{v^3 - 32000}\mathrm{d}v = -\int \mathrm{d}x$<br>$160 \ln(v^3 - 32000) = -x \quad (+A)$<br><br>$160 \ln(v^3 - 32000) = -x + 160 \ln 32000$<br>or<br>$160 \ln(v^3 - 32000) - 160 \ln 32000 = -500$<br><br>$(v^3 - 32000)/32000 = e^{-x/160}$<br>Speed of $m/c$ is $32.2\mathrm{ms}^{-1}$ | M1<br><br>A1<br><br>M1<br><br>A1ft<br><br>B1ft<br>B1<br>[6] | For separating variables and integrating<br><br>For using $v(0) = 40$ or<br>$[160 \ln(v^3 - 32000)]_{40}^{v} = [-x]_{0}^{500}$<br><br>ft where factor 160 is incorrect but +ve,<br><br>Implied by $(v^3 - 32000)/32000 = e^{-3.125}$ (or = 0.0439 ..). ft where factor 160 is incorrect but +ve, or for an incorrect non-zero value of $A$ |
| **5 i** | $x_{max} = \sqrt{1.5^2 + 2^2} - 1.5 \ (= 1)$<br>$[T_{max} = 18 \times 1/1.5]$<br>Maximum tension is 12N | B1<br>M1<br>A1<br>[3] | For using $T = \lambda x/L$ |
| | **(a)**<br><br>Gain in EE = $2[18(1^2 - 0.2^2)]/(2\times1.5)$ (11.52)<br><br>Loss in GPE = 2.8mg (27.44m) | M1<br><br>A1<br><br>B1 | For using EE = $\lambda x^2/2L$<br>May be scored with correct EE terms in expressions for total energy on release and total energy at lowest point<br>May be scored with correct GPE terms in expressions for total energy on release and total energy at lowest point |
| **ii** | $[2.8m \times 9.8 = 11.52]$<br>$m = 0.42$<br><br>**(b)**<br><br>$\frac{1}{2}mv^2 = mg(0.8) + 2\times18\times0.2^2/(2\times1.5)$ or<br>$\frac{1}{2}mv^2 = 2\times18\times1^2/(2\times1.5) - mg(2)$<br>Speed at $M$ is $4.24\mathrm{ms}^{-1}$ | M1<br>A1<br>[5]<br><br>M1<br><br>A1ft<br>A1ft<br>[3] | For using the p.c.energy<br>AG<br><br>For using the p.c.energy KE, PE & EE must all be represented<br>ft only when just one string is considered throughout in evaluating EE<br>ft only for answer 4.10 following consideration of only one string |

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| **6**<br>**i** | $[-mg\sin\theta = mL(d^2\theta/dt^2)]$<br>$d^2\theta/dt^2 = -(g/L)\sin\theta$ | M1<br>A1<br>[2] | For using Newton's second law tangentially with $a = Ld^2\theta/dt^2$<br>AG |
| **ii** | $[d^2\theta/dt^2 = -(g/L)\theta]$<br>$d^2\theta/dt^2 = -(g/L)\theta$ ➔ motion is SH | M1<br>A1<br>[2] | For using $\sin\theta \approx \theta$ because $\theta$ is small ($\theta_{max} = 0.05$)<br>AG |
| **iii** | $[4\pi/7 = 2\pi/\sqrt{9.8/L}]$<br>$L = 0.8$ | M1<br>A1<br>[2] | For using $T = 2\pi/n$ where $-n^2$ is coefficient of $\theta$ |
| **iv** | $[\theta = 0.05\cos 3.5\times 0.7]$<br>$\theta = -0.0385$<br><br>$t = 1.10$ (accept 1.1 or 1.09) | M1<br>A1ft<br><br>M1<br><br>A1ft<br>[4] | For using $\theta = \theta_o\cos nt$ {$\theta = \theta_o\sin nt$ not accepted unless the $t$ is reconciled with the $t$ as defined in the question}<br>ft incorrect $L$ {$\theta = 0.05\cos[4.9/(5L)^{1/2}]$}<br>For attempting to find $3.5t$ ($\pi < 3.5t < 1.5\pi$) for which $0.05\cos 3.5t$ = answer found for $\theta$ or for using $3.5(t_1 + t_2) = 2\pi$<br>ft incorrect $L$ {$t = [2\pi(5L)^{1/2}]/7 - 0.7$} |
| **v** | $\dot{\theta}^2 = 3.5^2(0.05^2 - (-0.0385)^2)$ or<br>$\dot{\theta} = -3.5\times 0.05\sin(3.5\times 0.7)$ ($\dot{\theta} = -0.1116..$)<br>Speed is $0.0893\,ms^{-1}$<br><br>(Accept answers correct to 2 s.f.) | M1<br><br><br><br>A1ft<br>A1ft<br>[3] | For using $\dot{\theta}^2 = n^2(\theta_o^2 - \theta^2)$ or $\dot{\theta} = -n\theta_o\sin nt$ {also allow $\dot{\theta} = n\theta_o\cos nt$ if $\theta = \theta_o\sin nt$ has been used previously}<br>ft incorrect $\theta$ with or without 3.5 represented by $(g/L)^{1/2}$ using incorrect $L$ in (iii) or for $\dot{\theta} = 3.5\times 0.05\cos(3.5\times 0.7)$ following previous use of $\theta = \theta_o\sin nt$<br>ft incorrect $L$ ($L\times 0.089287/0.8$ with $n = 3.5$ used or from $\lvert 0.35\sin\{4.9/[5L]^{1/2}\}/[5L]^{1/2}\rvert$ |
| | | | **SR** for candidates who use $\dot{\theta}$ as $v$. (Max 1/3)<br>For $v = \pm 0.112$ B1 |

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| **7 i** | Gain in PE = $mga(1 - \cos\theta)$<br>[$\frac{1}{2} mu^2 - \frac{1}{2} mv^2 = mga(1 - \cos\theta)$ ] | B1<br>M1 | <br>For using KE loss = PE gain |
| | $v^2 = u^2 - 2ga(1 - \cos\theta)$<br>[$R - mg\cos\theta = m$(accel.)]<br>$R = mv^2/a + mg\cos\theta$<br><br>[$R = m\{u^2 - 2ga(1 - \cos\theta)\}/a + mg\cos\theta$ ]<br>$R = mu^2/a + mg(3\cos\theta - 2)$ | A1<br><br>M1<br>A1<br>M1<br>A1<br>[7] | <br><br>For using Newton's second law radially<br><br>For substituting for $v^2$<br>AG |
| **ii** | [$0 = mu^2/a - 5mg$]<br>$u^2 = 5ag$<br><br><br>[$v^2 = 5ag - 4ag$]<br>Least value of $v^2$ is $ag$ | M1<br>A1<br><br><br>M1<br>A1<br>[4] | For substituting $R = 0$ and $\theta = 180^\circ$<br><br>For substituting for $u^2$ (= $5ag$) and $\theta$ = $180^\circ$ in $v^2$ (expression found in (i)) { but M0 if<br>$v = 0$ has been used to find $u^2$}<br>AG |
| **iii** | [$0 = u^2 - 2ga(1 - \frac{\sqrt{3}}{2})$]<br>$u^2 = ag(2 - \sqrt{3})$ | M1<br><br>A1<br>[2] | For substituting $v^2 = 0$ and $\theta = \pi/6$ in $v^2$ (expression found in (i))<br><br>Accept $u^2 = 2ag(1 - \cos\pi/6)$ |

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# 4731 Mechanics 4

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| **1 (i)** | Using $\omega_2^2 = \omega_1^2 + 2\alpha\theta$, $67^2 = 83^2 + 2\alpha \times 1000$<br>$\alpha = -1.2$<br>Angular deceleration is $1.2\ \text{rad s}^{-2}$ | M1<br>A1<br>**[2]** | |
| **(ii)** | Using $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$,<br>$400 = 83t - 0.6t^2$<br>$t = 5$ *or* $133\frac{1}{3}$<br>Time taken is 5 s | M1<br>A1ft<br>M1<br>A1<br>**[4]** | Solving to obtain a value of $t$ |
| | *Alternative for (ii)*<br>$\omega_2^2 = 83^2 - 2\times 1.2\times 400$ M1A1 ft<br>$\omega_2 = 77$<br>$77 = 83 - 1.2t$ M1<br>$t = 5$ A1 | | *( M0 if $\omega = 67$ is used in (ii) )* |

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| **2** | Volume $V = \int \pi y^2\,\mathrm{d}x = \int_a^{2a} \pi \frac{a^6}{x^4}\mathrm{d}x$<br>$= \pi\left[-\frac{a^6}{3x^3}\right]_a^{2a} = \frac{7}{24}\pi a^3$<br>$V\bar{x} = \int \pi x y^2\,\mathrm{d}x$<br>$= \int_a^{2a} \pi \frac{a^6}{x^3}\mathrm{d}x$<br>$= \pi\left[-\frac{a^6}{2x^2}\right]_a^{2a} = \frac{3}{8}\pi a^4$<br>$\bar{x} = \frac{\frac{3}{8}\pi a^4}{\frac{7}{24}\pi a^3}$<br>$= \frac{9a}{7}$ | M1<br>A1<br>M1<br>A1<br>A1<br>M1<br>A1<br>**[7]** | $\pi$ *may be omitted throughout*<br>For integrating $x^{-4}$ to obtain $-\frac{1}{3}x^{-3}$<br>for $\int xy^2\mathrm{d}x$<br>Correct integral form (including limits)<br>For integrating $x^{-3}$ to obtain $-\frac{1}{2}x^{-2}$<br>*Dependent on previous M1M1* |

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| **3 (i)** | $I = \frac{1}{2}(4m)(2a)^2 + (4m)a^2$<br>$+m(3a)^2$<br>$= 21ma^2$ | M1<br>A1<br>B1<br>A1<br>**[4]** | Applying parallel axes rule |
| **(ii)** | From P, $\bar{x} = \frac{(4m)a + m(3a)}{5m}$ $(= \frac{7a}{5})$<br>Period is $2\pi\sqrt{\frac{21ma^2}{5mg(\frac{7}{5}a)}}$<br>$= 2\pi\sqrt{\frac{3a}{g}}$ | M1<br>M1<br>A1 ft<br>A1<br>**[4]** | Correct formula $2\pi\sqrt{\frac{I}{mgh}}$ seen<br>*or* using $L = I\dot{\theta}$ and period $2\pi/\omega$ |
| | *Alternative for (ii)*<br>$-4mga\sin\theta - mg(3a)\sin\theta = (21ma^2)\ddot{\theta}$ M1<br>M1<br>Period is $2\pi\sqrt{\frac{21ma^2}{7mga}} = 2\pi\sqrt{\frac{3a}{g}}$ A1 ft A1 | | Using $L = I\dot{\theta}$ with three terms<br>Using period $2\pi/\omega$ |

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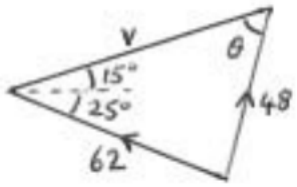
| **4 (i)** |  <br>$\frac{\sin\theta}{62}=\frac{\sin 40}{48}$<br>$\theta = 56.1°$ *or* $123.9°$<br>Bearings are $018.9°$ and $311.1°$ | M1<br>M1<br>A1<br>A1A1<br>[5] | Velocity triangle<br><br>One value sufficient<br>Accept $19°$ and $311°$ |
| **(ii)** | Shorter time when $\theta = 56.1°$<br>$\frac{v}{\sin 83.87}=\frac{48}{\sin 40}$<br>Relative speed is $v = 74.25$<br>Time to intercept is $\frac{3750}{74.25}$<br>$= 50.5$ s | B1 ft<br>M1<br>M1<br>A1<br>[4] | <br>Or $v^2 = 62^2 + 48^2 - 2\times 62\times 48\cos 83.87$<br>*Dependent on previous M1*<br> |
| | *Alternative for (i) and (ii)*<br>$\begin{pmatrix}48\sin\phi\\48\cos\phi\end{pmatrix}t=\begin{pmatrix}3750\sin 75\\3750\cos 75\end{pmatrix}+\begin{pmatrix}62\sin 295\\62\cos 295\end{pmatrix}t$ M1<br><br>$3.732\cos\phi-\sin\phi=3.208$ A1<br><br><br><br>$\phi = 18.9°$ and $311.1°$ A1A1<br><br>$t = 50.5$ A1 | <br><br><br>

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| **5 (i)** | Area is $\int_0^2 (8-x^3)\,\mathrm{d}x = \left[ 8x - \frac{1}{4}x^4 \right]_0^2 = 12$ | B1 | |
| | Mass per $\mathrm{m}^2$ is $\rho = \frac{63}{12} = 5.25$ | M1 | |
| | $I_y = \sum(\rho y\,\delta x)x^2 = \rho\int x^2 y\,\mathrm{d}x$ | M1 | for $\int x^2 y\,\mathrm{d}x$ or $\int x^3\,\mathrm{d}y$ |
| | $= \rho\int_0^2 (8x^2 - x^5)\,\mathrm{d}x$ | A1 | or $\frac{1}{3}\rho\int_0^8 (8-y)\,\mathrm{d}y$ |
| | $= \rho\left[ \frac{8}{3}x^3 - \frac{1}{6}x^6 \right]_0^2 = \frac{32}{3}\rho$ | A1 | for $\frac{32}{3}$ |
| | $= \frac{32}{3}\times\frac{63}{12} = 56\ \mathrm{kg\,m}^2$ | A1<br>AG<br>**[6]** | |
| **(ii)** | Anticlockwise moment is $800 - 63\times 9.8\times\frac{4}{5}$<br>$= 306.08\ \mathrm{N\,m} > 0$<br>so it will rotate anticlockwise | M1<br><br>A1<br><br>**[2]** | Full explanation is required; (anti)clockwise should be mentioned before the conclusion |
| **(iii)** | $I = I_x + I_y = 1036.8 + 56 \quad (= 1092.8)$ | B1 | |
| | WD by couple is $800\times\frac{1}{2}\pi$ | B1 | |
| | Change in PE is $63\times 9.8\times\left(\frac{24}{7} - \frac{4}{5}\right)$ | B1 | |
| | $800\times\frac{1}{2}\pi = \frac{1}{2}I\omega^2 - 63\times 9.8\times\left(\frac{24}{7} - \frac{4}{5}\right)$ | M1 | Equation involving WD, KE and PE |
| | $1256.04 = 546.4\,\omega^2 - 1622.88$ | A1 | *May have an incorrect value for I; other terms and signs are cao* |
| | $\omega = 2.30\ \mathrm{rad\,s}^{-1}$ | A1<br>**[6]** | |

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| **6 (i)** | GPE is $mg(a\sin 2\theta)$<br>$AB = 2a\cos\theta$ *or* $AB^2 = a^2 + a^2 - 2a^2\cos(\pi - 2\theta)$<br>EPE is $\frac{\sqrt{3}mg}{2a}(2a\cos\theta)^2$<br>$= \sqrt{3}mga(1+\cos 2\theta)$<br>Total PE is $V = \sqrt{3}mga(1+\cos 2\theta) + mga\sin 2\theta$<br>$= mga(\sqrt{3} + \sqrt{3}\cos 2\theta + \sin 2\theta)$ | B1<br><br><br>B1<br><br>M1<br><br><br>A1<br>AG<br>[4] | Or $mg(2a\cos\theta\sin\theta)$<br><br><br>Any correct form<br><br>Expressing EPE and GPE in terms of $\cos 2\theta$ and $\sin 2\theta$ |
| **(ii)** | $\frac{dV}{d\theta} = mga(-2\sqrt{3}\sin 2\theta + 2\cos 2\theta)$<br>$= 0$ when $2\sqrt{3}\sin 2\theta = 2\cos 2\theta$<br>$\tan 2\theta = \frac{1}{\sqrt{3}}$<br>$\theta = \frac{\pi}{12}, -\frac{5\pi}{12}$ | B1<br>M1<br><br><br>M1<br>A1A1<br>[5] | ( B0 for $\frac{dV}{d\theta} = -2\sqrt{3}\sin 2\theta + 2\cos 2\theta$ )<br><br><br><br>Solving to obtain a value of $\theta$<br>Accept $0.262, -1.31$ or $15°, -75°$ |
| **(iii)** | $\frac{d^2V}{d\theta^2} = mga(-4\sqrt{3}\cos 2\theta - 4\sin 2\theta)$<br>When $\theta = \frac{\pi}{12}$, $\frac{d^2V}{d\theta^2} = -8mga < 0$<br>so this position is unstable<br>When $\theta = -\frac{5\pi}{12}$, $\frac{d^2V}{d\theta^2} = 8mga > 0$<br>so this position is stable | B1ft<br><br>M1<br><br>A1<br><br><br>A1<br>[4] | <br><br>Determining the sign of $V''$ *or* M2 for alternative method for max / min |

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| **7 (i)** | Initially $\cos\theta = \frac{0.6}{1.5} = 0.4$<br>$\frac{1}{2}\times 4.9\,\omega^2 = 6\times 9.8(0.5\times 0.4 - 0.5\cos\theta)$<br>$\omega^2 = 12(0.4 - \cos\theta)$<br>$\omega^2 = 4.8 - 12\cos\theta$ | M1<br>A1<br><br>A1<br>AG<br>**[3]** | Equation involving KE and PE |
| **(ii)** | $6\times 9.8\times 0.5\sin\theta = 4.9\alpha$<br>$\alpha = 6\sin\theta$ (rad s$^{-2}$) | M1<br>A1<br>**[2]** | or $2\omega\frac{d\omega}{d\theta} = 12\sin\theta$ or $2\omega\frac{d\omega}{dt} = 12\sin\theta\frac{d\theta}{dt}$ |
| **(iii)** | $6\times 9.8\cos\theta - F = 6\times 0.5\,\omega^2$<br>$58.8\cos\theta - F = 14.4 - 36\cos\theta$<br>$F = 94.8\cos\theta - 14.4$<br><br>$6\times 9.8\sin\theta - R = 6\times 0.5\,\alpha$<br>$58.8\sin\theta - R = 18\sin\theta$<br>$R = 40.8\sin\theta$ | M1<br>M1<br><br>A1<br>AG<br>M1<br>M1<br>A1<br>**[6]** | for radial acceleration $r\omega^2$<br>radial equation of motion<br>*Dependent on previous M1*<br><br>for transverse acceleration $r\alpha$<br>transverse equation of motion<br>*Dependent on previous M1* |
| **(iv)** | If $B$ reaches the ground, $\cos\theta = -0.4$<br>$F = -52.32$<br>$\sin\theta = \sqrt{0.84}$ [ $\theta = 1.982$ *or* $113.6°$ ] $R = 37.39$<br>Since $\frac{52.32}{37.39} = 1.40 > 0.9$, this is not possible | M1<br>A1<br>M1<br>A1<br>**[4]** | *Allow M1A0 if* $\cos\theta = +0.4$ *is used*<br><br>Obtaining a value for $R$<br>Or $\mu R = 33.65$, and $52.32 > 33.65$ |
| | *Alternative for (iv)*<br>Slips when $F = -0.9R$<br>$94.8\cos\theta - 14.4 = -36.72\sin\theta$ M1<br>$\theta = 1.798$ [ $103.0°$ ] A1<br>$B$ reaches the ground when $\cos\theta = -0.4$ M1<br>$\theta = 1.982$ [ $113.6°$ ] so it slips before this A1 | | *Allow M1A0 if* $F = +0.9R$ *is used*<br><br>*Allow M1A0 if* $\cos\theta = +0.4$ *is used* |

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# 4732 Probability & Statistics 1

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| 1 | | | Q1: if consistent "0.8" incorrect or $^1/_8$, $^7/_8$ or 0.02 allow M marks in ii , iii & 1st M1 in i |
| i | Binomial stated<br><br>0.9437 – 0.7969 or $^8C_3 \times 0.2^3 \times 0.8^5$<br>= 0.147 (3 sfs) | M1<br><br>M1<br>A1 3 | or implied by use of tables or $^8C_3$<br>or $0.2^a \times 0.8^b$ $(a+b = 8)$ |
| ii | 1– 0.7969<br><br>= 0.203 (3 sf) | M1<br><br>A1 2 | allow 1– 0.9437 or 0.056(3)<br>or equiv using formula |
| iii | 8 × 0.2 oe<br>1.6 | M1<br>A1 2 | 8 × 0.2 = 2 M1A0<br>1.6 ÷ 8 or $^1/_{1.6}$ M0A0 |
| Total | | **7** | |
| 2 | first two $d$'s = ±1<br>$\Sigma d^2$ attempted (= 2)<br>$1 - \frac{6 \times \text{"2"}}{7(7^2-1)}$<br>$= {}^{27}/_{28}$ or 0.964 (3 sfs) | B1<br>M1<br>M1dep<br><br>A1 | $S_{xx}$ or $S_{yy}$ = 28 B1<br>$S_{xy}$ = 27 B1<br>$S_{xy}/\sqrt{(S_{xx}S_{yy})}$ M1 dep B1<br><br>1234567 & 1276543 (ans $^2/_7$): MR, lose A1 |
| Total | | **4** | |
| 3 i | $x$ independent or controlled or changed<br><br>Value of $y$ was measured for each $x$<br>$x$ not dependent | B1 1 | Allow Water affects yield, or yield is dependent<br>or yield not control water supply<br>Not just $y$ is dependent<br>Not $x$ goes up in equal intervals<br>Not $x$ is fixed |
| ii | (line given by) minimum<br>sum of squs | B1<br>B1 2 | B1 for "minimum" or "least squares" with inadequate or no explanation |
| iii | $S_{xx}$ = 17.5 or 2.92<br>$S_{yy}$ = 41.3 or 6.89<br>$S_{xy}$ = 25 or 4.17<br>$r = \frac{S_{xy}}{\sqrt{(S_{xx}S_{yy})}}$<br>= 0.930 (3 sf) | <br>B1<br><br>M1<br><br>A1 3 | or $91 - 21^2/_6$<br>or $394 - 46^2/_6$ B1 for any one<br>or $186 - {}^{21\times46}/_6$<br>dep B1<br><br>0.929 or 0.93 with or without wking B1M1A0<br>**SC** incorrect $n$: max B1M1A0 |
| iv | Near 1 or lg, high, strong, good corr'n or relnship oe<br><br>Close to st line or line good fit | B1ft<br><br>B1 2 | $\|r\|$ small: allow little (or no) corr'n oe<br><br>Not line accurate. Not fits trend |
| Total | | **8** | |

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| 4 | | | Q4: if consistent "0.7" incorrect or $^1/_3$, $^2/_3$ or 0.03 allow M marks in ii , iii & 1st M1 in i |
| i | Geo stated<br>$0.7^3 \times 0.3$ alone<br>$^{1029}/_{10000}$ or 0.103 (3 sf) | M1<br>M1<br>A1 3 | or implied by $q^n \times p$ alone ($n > 1$)<br>$0.7^3 - 0.7^4$ |
| ii | $0.7^4$ alone<br><br>$= {}^{2401}/_{10000}$ or 0.240 (3 sf) | M1<br><br>A1 2 | $1 - (0.3 + 0.7\times0.3 + 0.7^2\times0.3 + 0.7^3\times0.3)$<br>NB $1 - 0.7^4$ : M0 |
| iii | $1 - 0.7^5$<br><br><br><br>= 0.832 (3 sfs) | M2<br><br><br><br>A1 3 | or $0.3 + 0.7\times0.3 + + \ldots + 0.7^4\times0.3$ M2<br>M1 for one term extra or omitted or wrong or for 1– (above)<br>M1 for $1 - 0.7^6$ or $0.7^5$<br><br>NB Beware: $1 - 0.7^6 = 0.882$ |
| | | **8** | |
| 5i | $^{25}/_{10}$<br>= 2.5 | M1<br>A1 2 | Allow $^{25}/_{(9\text{to}10)}$ or 2.78: M1 |
| ii | (19.5, 25)<br>(9.5, 0) | B1<br>B1 2 | <br>Allow (24.5, 47)<br>Both reversed: SC B1<br>If three given, ignore (24.5, 47) |
| iii | Don't know exact or specific values of $x$ (or min or max or quartiles or median or whiskers). oe<br>Can only estimate (min or max or quartiles or median or whiskers) oe<br>Can't work out (…..) oe<br>Data is grouped oe | <br><br><br><br>B1 1 | Exact data not known<br><br>Allow because data is rounded |
| Total | | **5** | |

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| 6i | $\Sigma x \div 11$<br>70<br>$\Sigma x^2$ attempted<br>$\sqrt{\frac{\Sigma x^2}{11} - \bar{x}^2} = \sqrt{(^{54210}/_{11} - 70^2)}$ or $\sqrt{28.18}$ or 5.309<br>(= 5.31) **AG** | M1<br>A1<br>M1<br>A1<br>4 | $\geq 5$ terms, or $\Sigma(x-\bar{x})^2$<br>or $\sqrt{\frac{\Sigma(x-\bar{x})^2}{11}} = \sqrt{^{310}/_{11}}$ or $\sqrt{28.18}$<br>ie correct substn or result<br>If $\times\ ^{11}/_{10}$: M1A1M1A0 |
| ii | Attempt arrange in order<br>med = 67<br>74 and 66<br>IQR = 8 | M1<br>A1<br>M1<br>A1 4 | or (72.5 – 76.5) – (65.5 – 66.5 ) incl<br>must be from 74 – 66 |
| | | | iii, iv & v: ignore extras |
| iii | no (or fewer) extremes this year oe<br>sd takes account of all values<br>sd affected by extremes<br>less spread tho' middle 50% same<br>less spread tho' 3rd & 9th same or same gap | B1 1 | fewer high &/or low scores<br>highest score(s) less than last year<br>Not less spread or more consistent<br>Not range less |
| iv | sd measures spread or variation or consistency oe | B1 1 | sd less means spread is less oe<br>or marks are closer together oe |
| v | more consistent, more similar,<br>closer together, nearer to mean<br>less spread | B1 1 | allow less variance<br>Not range less<br>Not highest & lowest closer |
| Total | | **11** | |
| 7i | $^8C_3$<br>= 56 | M1<br>A1 2 | |
| ii | $^7C_2$ or or $^7P_2$ / $^8P_3$<br>$\div(^8C_3$ or "56") only<br>$= {}^3/_8$ | M1<br>M1<br>A1 3 | $^8C_1+{}^7C_1+{}^6C_1$ or 21<br>or $8\times7\times6$<br>or $^{\cdot\cdot}/_8 \times {}^{\cdot\cdot}/_7 \times {}^{\cdot\cdot}/_6$<br>indep, dep ans < 1 |
| | $^1/_8$ not from incorrect<br>$\times$ 3 only<br>or<br>$^1/_8 + {}^7/_8 \times {}^1/_7 + {}^7/_8 \times {}^6/_7 \times {}^1/_6$ | | $^7/_8 \times {}^6/_7 \times {}^5/_6$<br>1 – prod 3 probs |
| iii | $^8P_3$ or $8\times7\times6$ or $^8C_1 \times {}^7C_1 \times {}^6C_1$ or 336<br>$1 \div {}^8P_3$ only<br>$= {}^1/_{336}$ or 0.00298 (3 sf) | M1<br>M1<br>A1 3 | $^1/_8 \times {}^1/_7 \times {}^1/_6$ only M2 If $\times$ or $\div$: M1<br>$(^1/_8)^3$ M1 |
| Total | | **8** | |

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| 8ia | $^{18}/_{19}$ or $^{1}/_{19}$ seen<br>$^{17}/_{18}$ or $^{1}/_{18}$ seen<br>structure correct ie 6 branches<br><br>all correct incl. probs and W & R | B1<br>B1<br>B1<br><br>B1 4 | <br><br>regardless of probs & labels<br>(or 14 branches with correct 0s & 1s) |
| b | $^{1}/_{20} + {}^{19}/_{20} \times {}^{1}/_{19} + {}^{19}/_{20} \times {}^{18}/_{19} \times {}^{1}/_{18}$<br><br>$= {}^{3}/_{20}$ | M2<br><br>A1 3 | M1 any 2 correct terms added<br>$^{19}/_{20}\times{}^{18}/_{19}\times{}^{17}/_{18}$<br>$1 - {}^{19}/_{20}\times{}^{18}/_{19}\times{}^{17}/_{18}$ |
| iia | $^{19}/_{20} \times {}^{18}/_{19}$<br>$= {}^{9}/_{10}$ oe | M1<br>A1 2 | $^{19}/_{20}\times{}^{18}/_{19}\times{}^{1}/_{18}+{}^{19}/_{20}\times{}^{18}/_{19}\times{}^{17}/_{18}$ or $^{1}/_{20}+{}^{17}/_{20}$ |
| b | (P($X$ = 1) = $^{1}/_{20}$)<br>$^{19}/_{20} \times {}^{1}/_{19}$<br>$= {}^{1}/_{20}$<br><br>$\Sigma xp$<br>$= {}^{57}/_{20}$ or 2.85 | <br>M1<br>A1<br><br>M1<br>A1 4 | <br>or $1 - ({}^{1}/_{20} + {}^{9}/_{10})$<br>or 2 probs of $^{1}/_{20}$ M1A1<br><br>$\geq$ 2 terms, ft their $p$'s if $\Sigma p = 1$<br><br>NB: $^{19}/_{20}\times 3 = 2.85$ no mks |
| | | | With replacement: |
| ia | | | Original scheme |
| ib | | | $^{1}/_{20} + {}^{19}/_{20} \times {}^{1}/_{20} + ({}^{19}/_{20})^2 \times {}^{1}/_{20}$<br>or $1 - ({}^{19}/_{20})^2$ M1 |
| iia | | | $({}^{19}/_{20})^2$ or $({}^{19}/_{20})^2\times{}^{1}/_{20} + ({}^{19}/_{20})^2\times{}^{19}/_{20}$ M1 |
| b | | | Original scheme<br>But NB ans 2.85(25…) M1A0M1A0 |
| Total | | **13** | |

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|---|---|---|---|---|
| 9i | $(1 - 0.12)^n$<br>$\underline{\log 0.05}$<br>$\log 0.88$<br><br>$n = 24$ | <br>or $0.88^{23} = 0.052...$<br>or $0.88^{24} = 0.046...$ | M1<br><br>M1<br><br>A1 3 | Can be implied by 2nd M1 allow $n - 1$<br><br>or $\log_{0.88}0.05$ or 23.4(…)<br><br>Ignore incorrect inequ or equals signs |
| ii | ${}^6C_2 \times 0.88^4 \times 0.12^2$<br><br><br><br><br>$\times 0.12$<br>$= 0.0155$ | (= 0.1295… ) | M3<br><br><br><br><br>M1<br>A1 5 | or $0.88^4 \times 0.12^2$ M2<br>or ${}^6C_2 \times 0.88^4 \times 0.12^2$ + extra M2<br><br>or 2 successes in 6 trials implied<br>or ${}^6C_2$ M1<br><br>dep ≥ M1<br><br>$0.88^4 \times 0.12^2 \times 0.12$: M2M1<br>$0.88^4 \times 0.12^3$ M0M0A0<br>unless clear P(2 success in 6 trials) × 0.12<br>in which case M2M1A0 |
| Total | | | **8** | |

**Total 72 marks**

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# 4733 Probability & Statistics 2

| | | | |
|---|---|---|---|
| **1** | $\frac{105.0-\mu}{\sigma} = -0.7; \frac{110.0-\mu}{\sigma} = -0.5$<br><br>Solve:<br>$\sigma = 25$<br>$\mu = 122.5$ | M1<br>A1<br><br>B1<br>M1<br>A1<br>A1 **6** | Standardise once, equate to $\Phi^{-1}$, allow $\sigma^2$<br>Both correct including signs & $\sigma$, no cc (continuity correction), allow wrong $z$<br>Both correct $z$-values. "1 –" errors: M1A0B1<br>Get either $\mu$ or $\sigma$ by solving simultaneously<br>$\sigma$ a.r.t. 25.0<br>$\mu = 122.5 \pm 0.3$ or 123 if clearly correct, allow from $\sigma^2$ but *not* from $\sigma = -25$. |
| **2** | Po(20) ≈ N(20, 20)<br>Normal approx. valid as $\lambda > 15$<br>$1-\Phi\left(\frac{24.5-20}{\sqrt{20}}\right) = 1 - \Phi(1.006)$<br>$= 1 - 0.8427$ **= 0.1573** | M1<br>A1<br>B1<br>M1<br>A1<br>A1 **6** | Normal stated or implied<br>(20, 20) or (20, √20) or (20, $20^2$), can be implied<br>"Valid as $\lambda > 15$", *or* "valid as λ large"<br>Standardise 25, allow wrong or no cc, √20 errors<br>$1.0 < z \leq 1.01$<br>Final answer, art 0.157 |
| **3** | $H_0 : p = 0.6$, $H_1 : p < 0.6$<br>where $p$ is proportion in population who believe it's good value<br>$R \sim B(12, 0.6)$<br>α: $P(R \leq 4) = 0.0573$<br>$> 0.05$ | B2<br><br>M1<br>A1<br>B1 | Both, B2. Allow $\pi$, %<br>One error, B1, except $x$ or $\bar{x}$ or $r$ or $R$: 0<br>B(12, 0.6) stated or implied, e.g. N(7.2, 2.88)<br>*Not* P(< 4) or P(≥ 4) or P(= 4)<br>Must be using P(≤ 4), or P(> 4) < 0.95 and binomial |
| | β: CR is ≤ 3 and 4 > 3<br>$p = 0.0153$ | B1<br>A1 | Must be using CR; explicit comparison needed |
| | Do not reject $H_0$. Insufficient evidence that the proportion who believe it's good value for money is less than 0.6 | M1<br>A1 **7** | Correct conclusion, needs B(12,0.6) and ≤ 4<br>Contextualised, some indication of uncertainty<br>[**SR**: N(7.2, …) or Po(7.2): poss B2 M1A0]<br>[**SR**: P(< 4) or P(= 4) or P(≥ 4): B2 M1A0] |
| **4** (i) | Eg "not all are residents";<br>"only those in street asked" | B1<br>B1 **2** | One valid relevant reason<br>A definitely different valid relevant reason<br>*Not* "not a random sample", *not* "takes too long" |
| (ii) | Obtain list of whole population<br>Number it sequentially<br>Select using random numbers<br>[Ignore method of making contact] | B1<br>B1<br>B1 **3** | "Everyone" or "all houses" must be implied<br>*Not* "number it with random numbers" unless then "arrange in order of random numbers"<br>**SR**: "Take a random sample": B1<br>**SR**: Systematic: B1 B0, B1 if start randomly chosen |
| (iii) | Two of: α: Members of population equally likely to be chosen<br>β: Chosen independently/randomly<br>γ: Large sample (e.g. > 30) | B1<br><br>B1 **2** | One reason. NB : If "independent", must be "chosen" independently, not "views are independent"<br>Another reason. Allow "fixed sample size" but not both that and "large sample". Allow "houses" |

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|---|---|---|---|---|
| **5** | (i) | Bricks scattered at constant average rate & independently of one another | B1<br>B1 **2** | B1 for each of 2 different reasons, in context.<br>(Treat "randomly" ≡ "singly" ≡ "independently") |
| | (ii) | Po(12)<br>P(≤ 14) – P(≤ 7) [= .7720 – .0895]<br>[or P(8) + P(9) + … + P(14)]<br>**= 0.6825** | B1<br>M1<br><br><br>A1 **3** | Po(12) stated or implied<br>Allow one out at either end or both, eg 0.617, or wrong column, but *not* from Po(3) nor, eg, .9105 – .7720<br>Answer in range [0.682, 0.683] |
| | (iii) | $e^{-\lambda} = 0.4$<br>$\lambda = -\ln(0.4)$<br>= 0.9163<br>Volume = 0.9163 ÷ 3 = **0.305** | B1<br>M1<br>A1<br>M1 **4** | This equation, aef, can be implied by, eg 0.9<br>Take ln, or 0.91 by T & I<br>$\lambda$ art 0.916 or 0.92, can be implied<br>Divide their $\lambda$ value by 3<br>[**SR**: Tables, eg 0.9÷3: B1 M0 A0 M1] |
| **6** | (i) | 33.6<br>$\frac{115782.84}{100} - 33.6^2$ [= 28.8684]<br>$\times\frac{100}{99}$ **= 29.16** | B1<br>M1<br><br>M1<br>A1 **4** | 33.6 clearly stated [not recoverable later]<br>Correct formula used for biased estimate<br>$\times\frac{100}{99}$, M's independent. Eg $\frac{\Sigma r^2}{99}[-33.6^2]$<br>**SR** B1 variance in range [29.1, 29.2] |
| | (ii) | $\bar{R} \sim N(33.6, 29.16/9)$<br>$= N(33.6, 1.8^2)$<br>$1-\Phi\left(\frac{32-33.6}{\sqrt{3.24}}\right)$ [= Φ(0.8889)]<br>**= 0.8130** | M1<br>A1<br>M1<br><br>A1 **4** | Normal, their $\mu$, stated or implied<br>Variance [their (i)]÷9 [*not* ÷100]<br>Standardise & use Φ, 9 used, answer > 0.5, allow √ errors, allow cc 0.05 but *not* 0.5<br>Answer, art 0.813 |
| | (iii) | No, distribution of $R$ is normal so that of $\bar{R}$ is normal | B2 **2** | Must be saying this. Eg "9 is not large enough": B0. Both: B1 max, unless saying that $n$ is irrelevant. |
| **7** | (i) | $\frac{2}{9}\int_0^3 x^3(3-x)dx = \frac{2}{9}\left[\frac{3x^4}{4} - \frac{x^5}{5}\right]_0^3$ [= 2.7] –<br>$(1\frac{1}{2})^2$ $= \frac{9}{20}$ or **0.45** | M1<br>A1<br>B1<br>M1<br>A1 **5** | Integrate $x^2f(x)$ from 0 to 3 [*not* for μ]<br>Correct indefinite integral<br>Mean is 1½, soi [not recoverable later]<br>Subtract their $\mu^2$<br>Answer art 0.450 |
| | (ii) | $\frac{2}{9}\int_0^{0.5} x(3-x)dx = \frac{2}{9}\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^{0.5}$<br>$= \frac{2}{27}$ AG | M1<br><br>A1 **2** | Integrate f($x$) between 0, 0.5, must be seen somewhere<br>Correctly obtain given answer $\frac{2}{27}$, decimals other than 0.5 not allowed, 1 more line needed (eg [ ] = ⅓) |
| | (iii) | B(108, $\frac{2}{27}$)<br>≈ N(8, 7.4074)<br>$1-\Phi\left(\frac{9.5-8}{\sqrt{7.4074}}\right)$<br>= 1 – Φ(0.5511)<br>**= 0.291** | B1<br>M1<br>A1<br>M1<br><br>A1<br>A1 **6** | B(108, $\frac{2}{27}$) seen or implied, eg Po(8)<br>Normal, mean 8 …<br>… variance (or SD) 200/27 or art 7.41<br>Standardise 10, allow √ errors, wrong or no cc, needs to be using B(108,…)<br>Correct √ and cc<br>Final answer, art 0.291 |

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|---|---|---|---|---|
| | (iv) | $\bar{X} \sim N(1.5, \frac{1}{240})$ | B1<br>B1√<br>B1√ **3** | Normal NB: *not* part (iii)<br>Mean their $\mu$<br>Variance or SD (their 0.45)/108 [not (8, 50/729)] |
| **8** | (i) | $H_0 : \mu = 78.0$<br>$H_1 : \mu \neq 78.0$<br>$z = \frac{76.4 - 78.0}{\sqrt{68.9/120}} = -2.1115$<br>$> -2.576$ or $0.0173 > 0.005$ | B1<br>B1<br>M1<br>A1<br>B1 | Both correct, B2.<br>One error, B1, but $x$ or $\bar{x}$ : B0.<br>Needs $\pm(76.4 - 78)/\sqrt{(\sigma \div 120)}$, allow √ errors<br>art –2.11, or $p = 0.0173 \pm 0.0002$<br>Compare $z$ with (–)2.576, or $p$ with 0.005 |
| | | $78 \pm z\sqrt{(68.9/120)}$<br>$= 76.048$<br>$76.4 > 76.048$ | M1<br>A1√<br>B1 | Needs 78 and 120, can be – only<br>Correct CV to 3 sf, √ on $z$<br>$z = 2.576$ and compare 76.4, allow from $78 \leftrightarrow 76.4$ |
| | | Do not reject $H_0$. Insufficient evidence that the mean time has changed | M1<br><br><br>A1√ **7** | Correct comparison & conclusion, needs 120, "like with like", correct tail, $\bar{x}$ and $\mu$ right way round<br>Contextualised, some indication of uncertainty |
| | (ii) | $\frac{1}{\sqrt{68.9/n}} > 2.576$<br>$\sqrt{n} > 21.38$,<br>$n_{min} =$ **458**<br>Variance is estimated | IGNORE INEQUALITIES THROUGHOUT<br>M1<br><br>M1<br>A1<br><br>B1 **4** | Standardise 1 with $n$ and 2.576, allow √ errors, cc etc but *not* 2.326<br>Correct method to solve for $\sqrt{n}$ (*not* from $n$)<br>458 only (*not* 457), *or* 373 from 2.326, signs correct<br>Equivalent statement, allow "should use $t$". In principle nothing superfluous, but "variance stays same" B1 bod |

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**Specimen Answers**

**Question 4:** Part (i)

| | | |
|---|---|---|
| α | Takes too long/too slow | B0 |
| β | Interviewing people in the street isn't a random sample | B0 |
| γ | Many tourists so not representative | B1 |
| δ | Those who don't shop won't have their views considered | B1 |
| ε | Interviewers biased as to who they ask | B1 |
| ζ | Views influenced by views of others | B1 |

Part (ii)

| | | |
|---|---|---|
| α | Choose a random sample of the town and ask their opinion | B1 |
| β | Choose names at random from the town's phone book | B1 |
| γ | A random number machine determines which house numbers should be used, and every street should have the same proportion of residents interviewed | B0B0B1 |
| δ | Visit everyone door to door and give them a questionnaire | B1B0B0 |
| ε | Assign everyone a number and select randomly | B1B0B0 |
| ζ | Assign everyone a number and select using random numbers | B1B0B1 |
| η | Ditto + "ignoring numbers that don't correspond to a resident" | B1B1B1 |
| θ | Assign each eligible person a number and pick numbers from a hat | B1B1B0 |
| ι | Put names of all residents into a hat and pick them out [NB: postal survey is biased] | B1B1B0 |

Part (iii)

| | | |
|---|---|---|
| α | One person's view should not affect another's | B0 |
| β | It is without bias | B0 |
| γ | Results occur randomly | B0 |
| δ | Should be asked if they are for or against (binomial testing) | B0 |
| ε | It will survey a diverse group from different areas so should be representative | B0 |
| ζ | Everyone's should be chose independently of everyone else | B1 |
| η | The sample size must be large | B1 |
| θ | Participants are chosen at random and independently from one another [though η & θ together would get B2] | B1 only |

**Question 5 (i)**

| | | | |
|---|---|---|---|
| α | Number of bricks must always be the same | | B0 |
| β | Results occur randomly | | B0 |
| γ | The chance of a brick being in one place is always the same | | B0 |
| δ | Events must occur independently and at constant average rate | | B0 |
| ε | They must occur independently and at constant average rate | | B1 only |
| ζ | Bricks' locations must be random and independent | *[effectively the same]* | B1 only |
| η | Only one brick in any one place; bricks independent | *[effectively the same]* | B1 only |

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# 4734 Probability & Statistics 3

Penalise 2 sf instead of 3 once only. Penalise final answer ≥ 6 sf once only.

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|---|---|---|---|---|
| **1** | (i) | $\int_0^1 \frac{2}{5}x^2\,dx + \int_1^4 \frac{2}{5}\sqrt{x}\,dx$<br>$= \left[\frac{2x^3}{15}\right]_0^1 + \left[\frac{4x^{3/2}}{15}\right]_1^4 = 2$ | M1<br><br>A1<br>A1 **3** | Attempt to integrate $xf(x)$, both parts added, limits<br><br>Correct indefinite integrals<br>Correct answer |
| | (ii) | $\int_2^4 \frac{2}{5\sqrt{x}}\,dx = \left[\frac{4\sqrt{x}}{5}\right]_2^4 = \frac{4}{5}(2-\sqrt{2})$ or 0.4686 | M1<br><br>A1<br>A1 **3** | Attempt correct integral, limits; needs "1 –" if $\mu < 1$<br>Correct indefinite integral, √ on their $\mu$<br>Exact aef, or in range [0.468, 0.469] |
| **2** | (i) | Po(0.5), Po(0.75)<br>Po(0.7) and Po(0.9)<br>$A + B \sim$ Po(1.6)<br><br>P($A + B \geq 5$) = 0.0237<br>B(20, 0.0237)<br>$0.9763^{20} + 20\times0.9763^{19}\times0.0237$<br>**= 0.9195** | M1<br>A1<br>M1<br><br>A1<br>M1<br>A1√<br>A1 **7** | 0.5, 0.75 scaled<br>These<br>Sum of Poissons used, can have wrong parameters<br>0.0237 from tables or calculator<br>Binomial (20, their $p$), soi<br>Correct expression, their $p$<br>Answer in range [0.919, 0.92] |
| | (ii) | Bacteria should be independent in drugs; *or* sample should be random | B1 **1** | Any valid relevant comment, must be contextualised |
| **3** | (i) | Sample mean = 6.486<br>$s^2 = 0.00073$<br>$6.486 \pm 2.776 \times \sqrt{\frac{0.00073}{5}}$<br>**(6.45, 6.52)** | B1<br>B1<br>M1<br><br>B1<br>A1A1 **6** | <br>0.000584 if divided by 5<br>Calculate sample mean ± $ts/\sqrt{5}$, allow 1.96, $s^2$ etc<br>$t = 2.776$ seen<br>Each answer, cwo (6.45246, 6.5195) |
| | (ii) | $2\pi \times$ above [= (40.5, 41.0)] | M1 **1** | |
| **4** | (i) | $H_0$: $p_1 = p_2$; $H_1$: $p_1 \neq p_2$, where $p_i$ is the proportion of all solvers of puzzle $i$<br>Common proportion 39/80<br>$s^2 = 0.4875 \times 0.5125 / 20$<br>$(\pm)\frac{0.6-0.375}{0.1117} = (\pm)2.013$<br><br>2.013 > 1.96, or 0.022 < 0.025<br>Reject $H_0$. Significant evidence that there is a difference in standard of difficulty | B1<br><br>M1A1<br>B1<br>M1<br>A1√<br><br>M1<br><br>A1√ **8** | Both hypotheses correctly stated, allow eg $\hat{p}$<br>[= 0.4875]<br>[= 0.01249, σ = 0.11176]<br>$(0.6 - 0.375)/s$<br>Allow 2.066√ from unpooled variance, $p$ = 0.0195<br>Correct method and comparison with 1.96 or 0.025, allow unpooled, 1.645 from 1-tailed *only*<br>Conclusion, contextualised, not too assertive |
| | (ii) | One-tail test used<br>Smallest significance level 2.2(1)% | M1<br>A1 **2** | One-tailed test stated or implied by Φ("2.013"), OK if off-scale; allow 0.022(1) |

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| **5** (i) | Numbers of men and women should have normal dists;<br>with equal variance;<br>distributions should be independent | B1<br><br>B1<br>B1 **3** | Context & 3 points: 2 of these, B1; 3, B2; 4, B3.<br>[Summary data: 14.73 49.06 52.57<br>16.24 62.18 66.07] |
| (ii) | $H_0: \mu_M = \mu_W$; $H_1: \mu_M \neq \mu_W$<br>$3992 - \frac{221^2}{15} + 5538 - \frac{276^2}{17}$ [≈ 1793]<br>1793/(14 + 16) = 59.766<br><br>$(\pm)\frac{221/15 - 276/17}{\sqrt{59.766(\frac{1}{15}+\frac{1}{17})}} = (-)0.548$<br><br>Critical region: $\lvert t \rvert \geq 2.042$<br>Do not reject $H_0$. Insufficient evidence of a difference in mean number of days | B1<br>M1<br>A1<br><br>A1<br><br>M1<br><br>A1√<br>A1<br><br>B1<br>M1<br><br>A1√ **10** | Both hypotheses correctly stated<br>Attempt at this expression (see above)<br>Either 1793 or 30<br>Variance estimate in range [59.7, 59.8] (or √ = 7.73)<br>Standardise, allow wrong (but not missing) $1/n$<br>Correct formula, allow $s^2(\frac{1}{15}+\frac{1}{17})$ or $(\frac{s_1^2}{15}+\frac{s_2^2}{17})$, allow 14 & 16 in place of 15, 17; 0.548 or –0.548<br>2.042 seen<br>Correct method and comparison type, must be *t*, allow 1-tail; conclusion, in context, not too assertive |
| (iii) | Eg Samples not indep't so test invalid | B1 **1** | Any relevant valid comment, eg "not representative" |

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|---|---|---|---|---|
| 6 | (i) | $F(0) = 0$, $F(\pi/2) = 1$<br>Increasing | B1<br>B1 **2** | Consider both end-points<br>Consider F between end-points, can be asserted |
| | (ii) | $\sin^4(Q_1) = ¼$<br>$\sin(Q_1) = 1/\sqrt{2}$<br>$Q_1 = \pi/4$ | M1<br>A1<br>A1 **3** | Can be implied. Allow decimal approximations<br>Or 0.785(4) |
| | (iii) | $G(y) = P(Y \le y) = P(T \le \sin^{-1} y)$<br>$= F(\sin^{-1} y)$<br>$= y^4$<br>$g(y) = \begin{cases} 4y^3 & 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$ | M1<br>A1<br>A1<br>M1<br>A1 **5** | Ignore other ranges<br>Differentiate G($y$)<br>Function and range stated, allow if range given in G |
| | (iv) | $\int_0^1 \frac{4}{1+2y}\,dy = [2\ln(1+2y)]_0^1$<br>$= \mathbf{2\ln 3}$ | M1<br>A1<br>A1 **3** | Attempt $\int \frac{g(y)}{y^3+2y^4}dy$ ; $\int_0^1 \frac{4}{1+2y}dy$<br>Or 2.2, 2.197 or better |
| 7 | (i)<br>α | $\Phi\left(\frac{8.084-8.592}{0.7534}\right) = \Phi(-0.674) = 0.25$<br>$\Phi(0) - \Phi(\text{above}) = 0.25$<br>$P(8.592 \le X \le 9.1) =$ same by symmetry | M1<br>A1<br>A1<br>A1 **4** | Standardise once, allow √ confusions, ignore sign<br>Obtain 0.25 for one interval<br>For a second interval, justified, eg using $\Phi(0) = 0.5$<br>For a third, justified, eg "by symmetry" |
| | *or*<br>β | $\frac{x-8.592}{0.7534} = 0.674$<br>$x = 8.592 \pm 0.674 \times 0.7534$<br>$= (8.084, 9.100)$ | M1A1<br>A1A1 | [from probabilities to ranges]<br>A1 for art 0.674 |
| | (ii) | $H_0$: normal distribution fits data<br>All E values 50/4 = 12.5<br>$X^2 = \frac{4.5^2+9.5^2+1.5^2+3.5^2}{12.5} = 10$<br>$10 > 7.8794$<br>Reject $H_0$.<br>Significant evidence that normal distribution is not a good fit. | B1<br>B1<br>M1<br>A1<br>B1<br>M1<br>A1√ **7** | *Not* N(8.592, 0.7534). Allow "it's normally distributed"<br>[Yates: 8.56: A0]<br>CV 7.8794 seen<br>Correct method, incl. formula for $\chi^2$ and comparison, allow wrong $\nu$<br>Conclusion, in context, not too assertive |
| | (iv) | $8.592 \pm 2.576 \times \frac{0.7534}{\sqrt{49}}$<br>(8.315, 8.869) | M1<br>A1<br>A1 **3** | Allow √ errors, wrong σ or $z$, allow 50<br>Correct, including $z = 2.576$ or $t_{49} = 2.680$, *not* 50<br>In range [8.31, 8.32] and in range (8.86, 8.87], even from 50, or (8.306, 8.878) from $t_{49}$ |

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# 4735 Probability & Statistics 4

| | | | |
|---|---|---|---|
| **1** | $M_{X_1+X_2}(t) = (e^{\mu_1 t+\frac{1}{2}\sigma_1^2 t^2})(e^{\mu_2 t+\frac{1}{2}\sigma_2^2})$<br>$= e^{(\mu_1+\mu_2)t+\frac{1}{2}(\sigma_1^2+\sigma_2^2)t^2}$ *oe*<br>$X_1 + X_2 \sim$ Normal distribution<br>with mean $\mu_1+\mu_2$, variance $\sigma_1^2+\sigma_2^2$ | M1<br>A1<br>A1<br>A1A1 **5**<br>**{5}** | MGF of sum of independent RVs<br><br>No suffices:- Allow M1A0A1A0A0 |
| **2** (i) | Non-parametric test used when the distribution of the variable in question is unknown | B1 **1** | |
| (ii) | $H_0: m_{V-A} = 0$, $H_1: m_{V-A} \neq 0$<br>where $m_{V-A}$ is the median of the population differences<br>Difference and rank, bottom up<br>$P = 65$ $Q = 13$<br>$T = 13$<br>Critical region: $T \leq 13$<br>13 is inside the CR so reject $H_0$ and accept that there is sufficient evidence at the 5% significance level that the medians differ<br>Use B(12, 0.5)<br>P( $\leq 4$) = 0.1938 or CR = {0,1,2,10,11,12}<br>$> 0.025$, accept that there is insufficient evidence, etc CWO | B1<br><br><br>M1<br>A1<br>B1<br>M1<br><br><br><br>A1<br>M1<br>A1<br>A1 **9** | Allow $m_V = m_A$ etc<br><br><br><br>Allow $P > Q$ stated<br><br><br><br><br>Penalise over-assertive conclusions once only.<br><br>Or 4 not in CR |
| (iii) | Wilcoxon test is more powerful than the sign test | B1 **1**<br>**{11}** | Use more information, more likely to reject NH |
| **3**(i) | $A + B$<br>$= \int_{-\infty}^{0} e^{2x} e^{xt} dx + \int_{0}^{\infty} e^{-2x} e^{xt} dx$<br>$= \left[\frac{1}{2+t} e^{(2+t)x}\right]_{-\infty}^{0} + \left[-\frac{1}{2-t} e^{-(2-t)x}\right]_{0}^{\infty}$<br>$= 1/(2 + t) + 1/(2 – t)$<br>$= 4/(4 – t^2)$ AG<br>$t < -2$, $A$ infinite; $t > 2$, B infinite | M1<br><br>B1 B1<br><br>A1<br>B1 **5** | Added, correct limits<br><br>Correct integrals<br><br>Allow sensible comments about denom of M($t$) |
| (ii) | **Either**: $4/(4 – t^2) = (1 – \frac{1}{4}t^2)^{-1}$<br>$= 1 + \frac{1}{4}t^2 + \ldots$ | M1<br>A1 | Expand |
| | **Or**: $M'(t) = 8t/(4 – t^2)^2$<br>$M''(t) = 8/(4 – t^2)^2 + t \times \ldots$ | | M1<br>A1 |
| | E($X$) = 0<br>Var($X$) = $2 \times \frac{1}{4} – 0 = \frac{1}{2}$ | M1<br>A1 **4**<br>**{9}** | For $M''(0) – [M'(0)]^2$ or equivalent<br>0.5 – 0 = 0.5 |

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| | | | |
|---|---|---|---|
| **4** (i) | G(1)=1 [$a+b=1$]<br>G′(1) = – 0.7 [$–a + 2b = – 0.7$]<br>Solve to obtain<br>$\boldsymbol{a = 0.9}$ , $\boldsymbol{b = 0.1}$ | M1<br>M1<br>M1<br>A1 **4** | |
| (ii) | G″($t$) [$=1.8/t^3 + 0.2$] and<br>G″(1) + G′(1) – [G′(1)²] used<br>Var = 2 – 0.7 – $0.7^2$ = **0.81** | M1<br><br>A1 **2** | |
| (iii) | $[(0.9 + 0.1t^3)/t]^{10}$<br>Method to obtain coefficient of $t^{-7}$<br>$10\times0.9^9\times0.1$<br>= **0.387** to 3SF | M1<br>M1<br>A1 ft<br>A1 **4**<br>**{10}** | $[(a + bt^3)/t]^{10}$<br>For both<br>Use of MGF. $10a^9b$ |
| **5** (i) | Marginal dist of $X_A$: 0.30 0.45 0.15 0.10<br>E = 0.45 + 0.3 + 0.3 = 1.05<br>Var = 0.45 + 0.6 + 0.9 – $1.05^2$<br>= **0.8475** | B1<br>B1<br><br>B1 **3** | |
| (ii) | Consider a particular case to show<br>P($X_A$ and $X_B$) ≠ P($X_A$)P($X_B$)<br>So $X_A$ and $X_B$ are not independent | M1<br><br>A1 **2** | Or E($X_A$), E($X_B$) and E($X_AX_B$)<br>1.05, 1.15, 1.09;<br>E($X_A$)E($X_B$) = 1.0275, ft on wrong E($X_A$) |
| (iii) | Cov = E($X_AX_B$) – E($X_A$)E($X_B$)<br>= 1.09 – 1.15×1.05 = –0.1175<br>Var($X_A – X_B$) = Var($X_A$) + Var($X_B$) –<br>2Cov($X_A$, $X_B$)<br>= **1.91** | M1<br>A1ft<br>M1<br>A1 **4** | Or from distribution of $X_A - X_B$<br>Wrong E($X_A$) |
| (iv) | Requires P($X_A$, $X_B$)/P($X_A+X_B = 1$)<br>= 0.13/(0.16 + 0.13)<br>= 13/29 = **0.448** | <br>M1<br>A1A1<br>A1 **4**<br>**{13}** | |

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| | | | |
|---|---|---|---|
| **6** (i) | $\int_a^\infty x e^{-(x-a)}\,dx = \left[-xe^{-(x-a)}\right]_a^\infty + \int_a^\infty e^{-(x-a)}\,dx$<br>$= a + [-e^{-(x-a)}]$<br>$= a + 1$ AG | M1B1<br><br>A1 **3** | Correct limits needed for M1; no, or incorrect, limits allowed for B1 |
| (ii) | $E(T_1) = (a+1) + 2(a+1) - 2(a+1) - 1$<br>$= a$<br>$E(T_2) = \frac{1}{4}(a+1+a+1) + (n-2)(a+1)/[2(n-2)] - 1$<br>$= a$<br>(So both are unbiased estimators of $a$) | M1<br>A1<br>M1<br>A1 **4** | |
| (iii) | $\sigma^2 = \text{Var}(X)$<br>$\text{Var}(T_1) = (1 + 4 + 1 + 1)\sigma^2 = 7\sigma^2$<br>$\text{Var}(T_2) = 2\sigma^2/16 + (n-2)\sigma^2/[2(n-2)^2]$<br>$= n\sigma^2/[8(n-2)]$ oe<br>This is clearly $< 7\sigma^2$, so $T_2$ is more efficient | M1<br>A1<br><br>B1<br>A1 **4** | |
| (iv) | eg $\frac{1}{n}(X_1 + X_2 + \ldots + X_n) - 1$ | B2 **2**<br>**{13}** | B1 for sample mean |
| **7** (i) | $D$ denotes "The person has the disease"<br>(a) $P(D) = p$, $P(D') = 1 - p$,<br>$P(+\|D) = 0.98$, $P(+\|D') = 0.08$<br>$P(+) = p \times 0.98 + 0.08 \times (1 - p)$<br>$= 0.08 + 0.9p$<br>$P(D\|+) = P(+\|D)(P(D)/P(+)$<br>$= 0.98p/(0.08 + 0.9p)$<br>(b) $P(D') \times P(+\|D') + P(D) \times P(-\|D)$<br>$= 0.08 - 0.06p$ | <br><br><br>M1<br><br>M1<br>A1<br>M1<br>A1 **5** | <br><br><br><br><br>Use conditional probability |
| (ii) | $P(++) = 0.98^2 \times p + 0.08^2 \times (1 - p)$<br>$P(D\|++) = 0.9604p/(0.954p + 0.0064)$ | M1<br>A1 **2** | |
| (iii) | Expected number with 2 tests:<br>$24000 \times 0.0809 = a$<br>Expected number with 1 test:<br>$24000 \times 0.9191 = b$<br>Expected total cost = £$(10a + 5b)$<br>= **£129 708** | <br>M1<br><br>M1<br>M1<br>A1 **4**<br>**{11}** | <br>Or: $0.08 + 0.9 \times 0.001$ oe<br><br>$\times 5 \times 24000$<br>$+5 \times 24000$ (dep 1st M1)<br>Or £130 000 |

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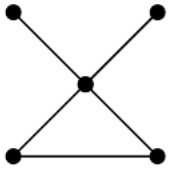
# 4736 Decision Mathematics 1

| | | | | |
|---|---|---|---|---|
| **1 (i)** | [43 172 536 17 314 462 220 231]<br><br>43 172 536 17 220<br>314 462<br>231 | <br><br>M1<br>M1<br>A1 | <br><br>First folder correct<br>Second folder correct<br>All correct (cao) | <br><br><br><br>[3] |
| **(ii)** | 536 462 314 231 220 172 43 17<br><br><br><br><br>536 462<br>314 231 220 172 43 17 | B1<br><br><br><br><br>M1<br>A1 | List sorted into decreasing order seen (cao)<br><br>[Follow through from a decreasing list with no more than 1 error or omission]<br><br>First folder correct<br>All correct | <br><br><br><br><br>[3] |
| **(iii)** | $(5000 \div 500)^2 \times 1.3$<br><br>= 130 seconds | M1<br><br>A1 | $10^2 \times 1.3$<br>or any equivalent calculation<br>Correct answer, with units | <br><br>[2] |
| | | | | **Total = 8** |

| | | | | |
|---|---|---|---|---|
| **2 (i)** | The sum of the orders must be even, (but 1+2+3+3 = 9 which is odd). | B1 | There must be an even number of odd nodes. | [1] |
| **(ii) a** | eg | M1<br><br>A1 | A graph with five vertices that is neither connected nor simple<br><br>Vertex orders 1, 1, 2, 2, 4 | <br><br>[2] |
| **b** | Because it is not connected | B1 | You cannot get from one part of the graph to the other part. | [1] |
| **c** | eg  | B1 | A connected graph with vertex orders 1, 1, 2, 2, 4 (Need not be simple) | [1] |
| **(iii) a** | There are five arcs joined to $A$.<br>Either Ann has met (at least) three of the others or she has met two or fewer, in which case there are at least three that she has not met.<br>In the first case at least three of the arcs joined to $A$ are blue, in the second case at least three of the arcs joined to $A$ are red. | M1<br><br><br><br>A1 | A reasonable attempt (for example, identifying that there are five arcs joined to $A$)<br><br>A convincing explanation (this could be a list of the possibilities or a well reasoned explanation) | <br><br><br><br>[2] |
| **b** | If any two of Bob, Caz and Del have met one another then $B$, $C$ and $D$ form a blue triangle with $A$. Otherwise $B$, $C$ and $D$ form a red triangle. | M1<br><br>A1 | A reasonable or partial attempt (using $A$ with $B$, $C$, $D$)<br>A convincing explanation (explaining both cases fully) | <br><br>[2] |
| | | | | **Total = 9** |

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| | | | | |
|---|---|---|---|---|
| **3**<br>**(i)** | $y \geq x$<br>$x + y \leq 8$<br>$x \geq 1$ | M1<br>M1<br>M1<br>A1 | Line $y = x$ in any form<br>Line $x + y = 8$ in any form<br>Line $x = 1$ in any form<br>All inequalities correct<br>[Ignore extra inequalities that do not affect the feasible region] | [4] |
| **(ii)** | (1, 1), (1, 7), (4, 4) | M1<br>A1 | Any two correct coordinates<br>All three correct<br>[Extra coordinates given ⇒ M1, A0] | [2] |
| **(iii)** | (1, 7) □ 23<br>(4, 4) □ 20<br>At optimum, $x = 1$ and $y = 7$<br>Maximum value = 23 | M1<br><br><br>A1<br>A1 | Follow through if possible<br>Testing vertices or using a line of constant profit (may be implied)<br>Accept (1, 7) identified<br>23 identified | [3] |
| **(iv)** | $2\times1 + k\times7 \geq 2\times4 + k\times4$<br><br>□ $k \geq 2$ | M1<br><br>A1 | $2 + 7k$ or implied, or using line of gradient $-\frac{2}{k}$<br>Greater than or equal to 2 (cao)<br>[$k > 2$ ⇒ M1, A0] | [2] |
| | | | | **Total = 11** |

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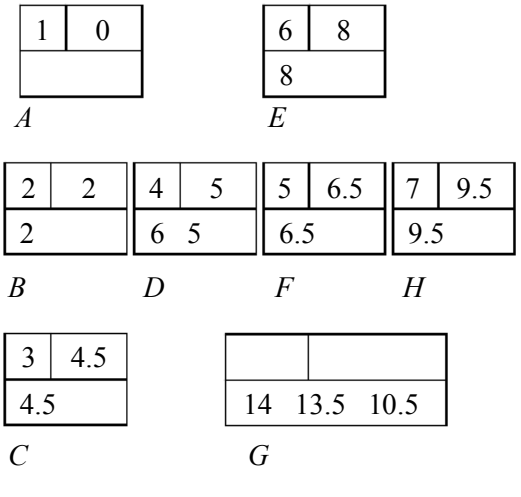
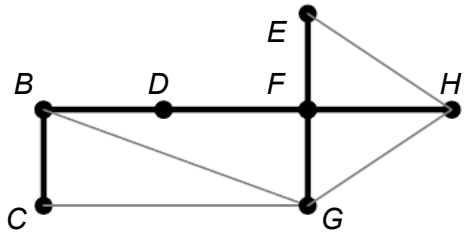
| | | | | |
|---|---|---|---|---|
| **4**<br>**(i)** |  <br>Route = *A* –*B* – *D* – *F* – *H*<br>Length = 9.5 miles | M1<br><br>M1<br>A1<br><br><br><br><br><br>B1<br><br><br>B1<br><br><br><br>B1<br>B1 | Both 6 and 5 shown at *D*<br>[5 may appear as perm label only]<br>14, 13.5 and 10.5 shown at *G*<br>No extra temporary labels<br>All temporary labels correct [condone perm values only appearing as perm labels]<br>[Dep on both M marks]<br><br>All permanent labels correct<br>[may omit *G*, but if given it must be correct]<br>Order of labelling correct<br>[may omit *G* but if given it must be correct]<br><br>cao<br>cao | [7] |
| **(ii)** | Route Inspection problem | B1 | Accept Chinese Postman | [1] |
| **(iii)** | Odd nodes: *A*, *D*, *E* and *H*<br>*AD* = 5 *AE* = 8 *AH* = 9.5<br>*EH* = 5 *DH* = 4.5 *DE* = 3.5<br>10 12.5 13.0<br><br>Repeat *AD* (*A*-*B*-*D*) and *EH* (*E*-*F*-*H*)<br>Length = 67.5 +10<br>= 77.5 miles | B1<br>M1<br>A1<br><br><br>M1<br><br>A1 | Identifying or using *A,D,E,H*<br>Attempting at least one pairing<br>At least one correct pairing or correct total<br><br><br>Adding their 10 to 67.5<br><br>77.5 (cao) | [5] |
| **(iv)** | Repeat arcs *EF* and *FD*<br>3.5 + 67.5 = 71 miles | B1<br>B1 | cao [ NOT *DE* or *D*-*F*-*E*]<br>cao | [2] |
| **(v)** | *A* – *B* – *C* – *G* –*F* – *D*<br>then method stalls<br>*E* and *H* are missed out | B1 | Showing route as far as *D* and then explaining the problem | [1] |
| **(vi)** | *C* – *B* – *A* – *D* – *F* – *E* – *H* – *G* – *C*<br><br>37.5 miles | M1<br>A1<br>B1 | [If final *C* is missing ⇒ M1, A0]<br>[A diagram needs arrows for A1]<br>37.5 (cao) | [3] |
| **(vii)** |  <br>Nodes: *B C D F E H G*<br>Weight = 16 miles<br><br>[Two shortest arcs from *A* are *AB* and *AD*]<br>2 + 6 + 16<br>Lower bound = 24 miles | M1<br><br>A1<br><br><br>B1<br>B1<br><br><br>M1<br>A1 | A spanning tree on reduced network (may show *AB*, *AD*)<br>Correct minimum spanning tree marked, with no extra arcs<br><br>cao<br>cao<br><br><br>8 + their 16 (or implied)<br>cao | [6] |
| | | | | **Total = 25** |

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**5 (i)**

$15x+15y+30z \leq 9000$

[divide through by 15 to get $x+y+2z \leq 600$ as given]

Stamping out: $5x+8y+10z \leq 3600$

Fixing pin: $50x+50y+50z \leq 25000$

$\Rightarrow x + y + z \leq 500$

Checking: $100x+50y+20z \leq 10000$

$\Rightarrow 10x+5y+2z \leq 1000$

B1 $15x+15y+30z \leq 9000$

B1 $5x+8y+10z \leq 3600$

B1 $x + y + z \leq 500$

B1 $10x+5y+2z \leq 1000$ **[4]**

**(ii)** $x$, $y$ and $z$ are non-negative

B1 $x \geq 0$, $y \geq 0$ and $z \geq 0$ **[1]**

**(iii)** ($P =$) $4x + 3y + z$

B1 cao [1]

**(iv)**

| $P$ | $x$ | $y$ | $z$ | $s$ | $t$ | $u$ | $v$ | RHS |
|---|---|---|---|---|---|---|---|---|
| 1 | -4 | -3 | -1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 2 | 1 | 0 | 0 | 0 | 600 |
| 0 | 5 | 8 | 10 | 0 | 1 | 0 | 0 | 3600 |
| 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 500 |
| 0 | 10 | 5 | 2 | 0 | 0 | 0 | 1 | 1000 |

B1 Follow through if reasonable
-4 -3 -1 in objective row

B1 Correct use of slack variables

M1 1 1 2 and 600 correct

A1 All constraint rows correct

Accept variations in order of rows and columns [4]

**(v)**

Pivot on the 10 in the $x$-column

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 0 | -1 | -0.2 | 0 | 0 | 0 | 0.4 | 400 |
| 0 | 0 | 0.5 | 1.8 | 1 | 0 | 0 | -0.1 | 500 |
| 0 | 0 | 5.5 | 9 | 0 | 1 | 0 | -0.5 | 3100 |
| 0 | 0 | 0.5 | 0.8 | 0 | 0 | 1 | -0.1 | 400 |
| 0 | 1 | 0.5 | 0.2 | 0 | 0 | 0 | 0.1 | 100 |

B1 Correct choice of pivot from $x$- column
[Follow through their tableau and valid pivot if possible: no negative values in RHS column and $P$ value has not decreased]

M1 Pivot row correct

A1 Other rows correct [3]

Pivot on 0.5 in the last row of $y$-column

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 0 | 0.2 | 0 | 0 | 0 | 0.6 | 600 |
| 0 | -1 | 0 | 1.6 | 1 | 0 | 0 | -0.2 | 400 |
| 0 | -11 | 0 | 6.8 | 0 | 1 | 0 | -1.6 | 2000 |
| 0 | -1 | 0 | 0.6 | 0 | 0 | 1 | -0.2 | 300 |
| 0 | 2 | 1 | 0.4 | 0 | 0 | 0 | 0.2 | 200 |

B1 Correct choice of pivot from $y$-column
[Follow through their tableau and valid pivot if possible]

M1 Pivot row correct

A1 Other rows correct [3]

$x = 0$, $y = 200$, $z = 0$, $P = 600$
Make 20 000 metallic badges (and no laminated badges or plastic badges)

B1 Interpretation of their $x$, $y$ and $z$ values in context (may imply zero entries)

To give a profit of £600

B1 Interpretation of their $P$ value in context

6000 seconds (100 min) of printing time not used, 2000 seconds (33 min 20 sec) of stamping out time not used, 15000 seconds (250 min) of fixing pin time not used.
All the checking time is used

B1 Interpretation of their slack variable values [3]

**Total = 19**

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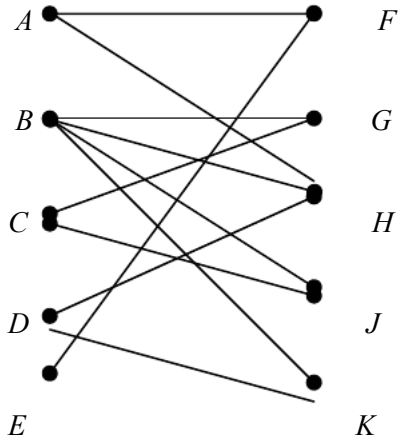
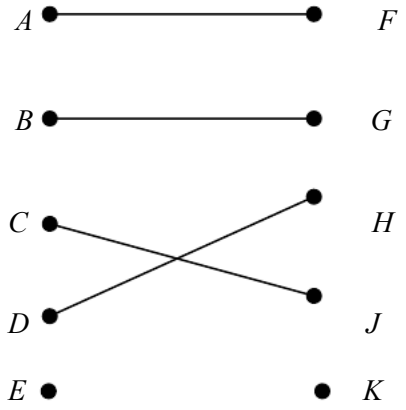
# 4737 Decision Mathematics 2

| | | | | |
|---|---|---|---|---|
| **1(a) (i)** |  | B1 | A correct bipartite graph | [1] |
| **(ii)** |  | B1 | A second bipartite graph showing the incomplete matching correctly | [1] |
| **(iii)** | $E = F - A = H - D = K$<br><br>Fiona = Egg and tomato $F = E$<br>Gwen = Beef and horseradish $G = B$<br>Helen = Avocado and bacon $H = A$<br>Jack = Chicken and stuffing $J = C$<br>Mr King = Duck and plum sauce $K = D$ | B1<br><br>B1 | This path in any reasonable form<br><br>This complete matching | [2] |
| **(iv)** | Interchange Gwen and Jack<br>$F = E \quad G = C \quad H = A \quad J = B \quad K = D$ | B1 | This complete matching | [1] |

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**(b)**

Reduce rows

| | *F* | *G* | *H* | *J* | *K* |
|---|---|---|---|---|---|
| *L* | 7 | 7 | 7 | 7 | 0 |
| *M* | 2 | 6 | 4 | 2 | 0 |
| *N* | 8 | 8 | 8 | 6 | 0 |
| *O* | 1 | 3 | 2 | 1 | 0 |
| *P* | 6 | 9 | 7 | 5 | 0 |

M1 Substantially correct attempt to reduce rows

Reduce columns

| | *F* | *G* | *H* | *J* | *K* |
|---|---|---|---|---|---|
| *L* | 6 | 4 | 5 | 6 | 0 |
| *M* | 1 | 3 | 2 | 1 | 0 |
| *N* | 7 | 5 | 6 | 5 | 0 |
| *O* | 0 | 0 | 0 | 0 | 0 |
| *P* | 5 | 6 | 5 | 4 | 0 |

M1 Substantially correct attempt to reduce columns

A1 cao

[3]

Cross out 0's using two (minimum no. of) lines

| | *F* | *G* | *H* | *J* | *K* |
|---|---|---|---|---|---|
| *L* | 6 | 4 | 5 | 6 | **0** |
| *M* | 1 | 3 | 2 | 1 | **0** |
| *N* | 7 | 5 | 6 | 5 | **0** |
| *O* | **0** | **0** | **0** | **0** | **0** |
| *P* | 5 | 6 | 5 | 4 | **0** |

Augment by 1

| | *F* | *G* | *H* | *J* | *K* |
|---|---|---|---|---|---|
| *L* | 5 | 3 | 4 | 5 | 0 |
| *M* | 0 | 2 | 1 | 0 | 0 |
| *N* | 6 | 4 | 5 | 4 | 0 |
| *O* | 0 | 0 | 0 | 0 | 1 |
| *P* | 4 | 5 | 4 | 3 | 0 |

M1 Substantially correct attempt at augmenting

A1 Augmenting correctly [2]

Cross out 0's using three (minimum no. of) lines

| | *F* | *G* | *H* | *J* | *K* |
|---|---|---|---|---|---|
| *L* | 5 | 3 | 4 | 5 | **0** |
| *M* | **0** | **2** | **1** | **0** | **0** |
| *N* | 6 | 4 | 5 | 4 | **0** |
| *O* | **0** | **0** | **0** | **0** | **1** |
| *P* | 4 | 5 | 4 | 3 | **0** |

Augment by 3

| | *F* | *G* | *H* | *J* | *K* |
|---|---|---|---|---|---|
| *L* | 2 | **0** | 1 | 2 | 0 |
| *M* | **0** | 2 | 1 | 0 | 3 |
| *N* | 3 | 1 | 2 | 1 | **0** |
| *O* | 0 | 0 | **0** | 0 | 4 |
| *P* | 1 | 2 | 1 | **0** | 0 |

M1 Substantially correct attempt at augmenting (by more than 1 in a single step)

A1 Augmenting correctly

Lemon = Gwen
Mandarin = Fiona
Nectarine = Mr King
Orange = Helen
Peach = Jack

B1 Correct allocation

[3]

**Total = 13**

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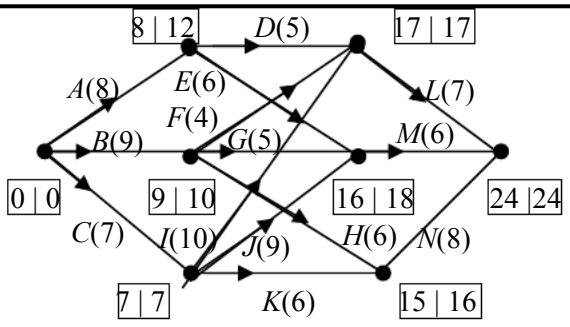
**2 (i)**

| Stage | State | Action | Working | Suboptimal maxima |
|---|---|---|---|---|
| 2 | 0 | 0 | 7 | 7 |
| | 1 | 0 | 6 | 6 |
| | 2 | 0 | 8 | 8 |
| 1 | 0 | 0 | 5 +7 = 12 | |
| | | 1 | 6 + 6 = 12 | 12 |
| | 1 | 0 | 4 +7 = 11 | |
| | | 1 | 5 + 6 = 11 | |
| | | 2 | 6 + 8 = 14 | 14 |
| | 2 | 0 | 10 + 7 = 17 | 17 |
| | | 1 | 9 + 6 = 15 | |
| | | 2 | 6 + 8 = 14 | |
| 0 | 0 | 0 | 8 + 12 = 20 | |
| | | 1 | 9 + 14 = 23 | |
| | | 2 | 7 + 17 = 24 | 24 |

Maximum route = (0;0) - (1;2) - (2;0) - (3;0)
Weight = 24

| | | |
|---|---|---|
| B1 | Structure of table correct | |
| M1 | Stage and state values correct | |
| A1 | Action values correct | [3] |
| B1 | Working backwards from stage 2<br>7, 6, 8 correct in suboptimal maxima column for stage 2 | |
| M1 | Working column substantially correct for stage 1 | |
| A1 | Sums correct for stage 1 | [3] |
| B1 | Suboptima maxima values correct for stage 1 | |
| M1 | Working column substantially correct for stage 0 | |
| A1 | Sums correct for stage 0 | [3] |
| B1 | Correct route from (0; 0) to (3; 0) | |
| B1 | 24 cao | [2] |

**(ii)**




Minimum completion time = 24
Critical activities: *C*, *I*, *L*

| | | |
|---|---|---|
| B1 | Assigning *A* to *N* appropriately | |
| M1 | Substantially correct forward pass | |
| A1 | Forward pass correct | |
| M1 | Substantially correct backward pass<br>Backward pass correct | |
| A1 | 24 (cao) | |
| B1 | *C*, *I*, *L* (cao) | |
| B1 | | [7] |

**(iii)**

The critical path is the maximum path
The critical activities form a continuous path with no slack, ie the longest path

| | | |
|---|---|---|
| M1 | Same path is found in both | |
| A1 | Recognition of why the solutions are the same, in general | [2] |

Total = **20**

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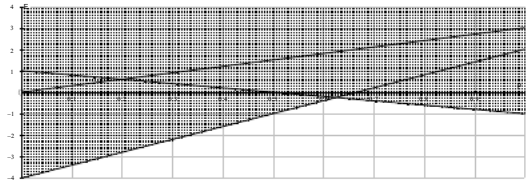
| | | | | |
|---|---|---|---|---|
| **3 (i)** | For each pairing, the total of the points is 10.<br>Subtracting 5 from each makes the total 0.<br><br>Eg 3 points and 7 points ⇒ scores of -2 and +2 | M1<br>A1<br><br> | Sum of points is 10<br>So sum of scores is zero<br><br>A specific example earns M1 only | [2] |
| **(ii)** | *W* scores -1<br>*P* has 6 points and *W* has 4 points | B1<br>B1 | -1<br>6 and 4 | [2] |
| **(iii)** | *W* is dominated by *Y*<br>-1 < 1, -3 < -2 and 1 < 2 | B1<br>B1 | *Y*<br>These three comparisons in any form | [2] |
| **(iv)** | Collies<br>*X* *Y* *Z* row min<br>Rovers *P* 2 -1 3 -1<br>*Q* 1 -3 -1 -3<br>*R* -4 1 0 -4<br>col max 2 1 3<br><br>Play-safe for Rovers is *P*<br>Play-safes for Collies is *Y* | M1<br><br><br><br><br>A1<br>A1 | Determining row minima and column maxima, or equivalent<br><br><br><br><br>*P*<br>*Y* | [3] |
| **(v)** | $2p - 4(1-p) = 6p - 4$<br>*Y* gives $1 - 2p$<br>*Z* gives $3p$ | B1<br><br>B1 | $6p - 4$ in simplified form<br><br>Both $1 - 2p$ and $3p$ in any form | [2] |
| **(vi)** |  <br><br>$6p - 4 = 1 - 2p \Rightarrow p = \frac{5}{8}$ | B1<br><br>M1<br><br>A1 | Their lines drawn correctly on a reasonable scale<br><br>Solving the correct pair of equations or using graph correctly<br>$\frac{5}{8}$, 0.625, cao | [3] |
| **(vii)** | Add 4 throughout matrix to make all values non-negative<br>On this augmented matrix,<br>if Collies play *X* Rovers expect $6p_1 + 5p_2$;<br>if Collies play *Y* Rovers expect $3p_1 + p_2 + 5p_3$;<br>and if Collies play *Z* Rovers expect $7p_1 + 3p_2 + 4p_3$<br><br>We want to maximise *M* where *M* only differs by a constant from *m* and, for each value of *p*, *m* is the minimum expected value. | B1<br><br>B1<br><br><br>B1 | 'Add 4', or new matrix written out or equivalent<br><br>Relating to columns *X*, *Y* and *Z* respectively. Note: expressions are given in the question.<br><br><br>For each value of *p* we look at the minimum output, then we maximise these minima. | [3] |
| **(viii)** | $p_3 = \frac{3}{8}$<br>$M = -\frac{1}{4}$ | B1<br><br>B1 | cao<br><br>cao | [2] |
| | | | **Total =** | **19** |

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| | | | | |
|---|---|---|---|---|
| **4 (i)** | 8+0+6+5+4<br>= 23 gallons per minute | M1<br>A1 | 8+0+6+5+4 or 23<br>23 with units | [2] |
| **(ii)** | At most 6 gallons per minute can enter $A$ so there cannot be 7 gallons per minute leaving it<br>At most 7 gallons per minute can leave $F$ so there cannot be 10 gallons per minute entering it. | B1<br><br>B1 | Maximum into $A$ = 6<br><br>Maximum out of $F$ = 7 | [2] |
| **(iii)** | A diagram showing a flow with 12 through $E$<br>Flow is feasible (upper capacities not exceeded)<br>Nothing flows through $A$ and $D$<br><br>Maximum flow through $E$ = 12 gallons per minute | M1<br>M1<br><br>A1<br><br>B1 | Assume that blanks mean 0<br><br><br><br>12 | [4] |
| **(iv) a** | If flows through $A$ but not $D$ its route must be $S – A – C – E$, but the flow through $E$ is already a maximum | B1 | A correct explanation | [1] |
| **b** | $S – (B) – C – D – F – T$<br>1 gallon per minute | M1<br>A1 | Follow through their part (iii)<br>1 | [2] |
| **(v)** | Flow = 12 + 1 = 13 gallons per minute<br><br>Cut through $ET$ and $FT$ or $\{S,A,B,C,D,E,F\}$, $\{T\}$<br>= 13 gallons per minute<br><br>Every cut forms a restriction<br>Every cut ≥ every flow □ min cut ≥ max flow<br><br>This cut = this flow<br>so must be min cut and max flow | <br><br>B1<br><br><br>M1<br>A1<br><br>B1 | <br><br>Identifying this cut in any way<br><br><br>Use of max flow – min cut theorem<br>min cut ≥ max flow<br><br>This cut = this flow (or having shown that both are 13) | [4] |
| **(vi)** | 3 gallons per minute<br>Must flow 6 along $ET$ and 7 along $FT$.<br>Can send 4 into $F$ from $D$ so only need to send 9 through $E$ | B1<br>B1<br>B1 | 3<br><br>A correct explanation | [3] |
| **(vii)** | A diagram showing a flow of 13 without using $BE$<br>Flow is feasible and only sends 9 through $E$ | M1<br><br>A1 | May imply directions and assume that blanks mean 0 | [2] |
| | | | **Total =** | **20** |

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# Grade Thresholds

**Advanced GCE Mathematics (3890-2, 7890-2)**
**June 2009 Examination Series**

**Unit Threshold Marks**

| 7892 | | Maximum Mark | A | B | C | D | E | U |
|---|---|---|---|---|---|---|---|---|
| 4721 | Raw | 72 | 58 | 51 | 44 | 38 | 32 | 0 |
| | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4722 | Raw | 72 | 56 | 49 | 42 | 35 | 28 | 0 |
| | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4723 | Raw | 72 | 53 | 46 | 39 | 33 | 27 | 0 |
| | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4724 | Raw | 72 | 53 | 46 | 39 | 33 | 27 | 0 |
| | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4725 | Raw | 72 | 49 | 43 | 37 | 32 | 27 | 0 |
| | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4726 | Raw | 72 | 53 | 46 | 40 | 34 | 28 | 0 |
| | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4727 | Raw | 72 | 55 | 49 | 43 | 38 | 33 | 0 |
| | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4728 | Raw | 72 | 62 | 52 | 42 | 33 | 24 | 0 |
| | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4729 | Raw | 72 | 57 | 48 | 39 | 31 | 23 | 0 |
| | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4730 | Raw | 72 | 61 | 51 | 41 | 32 | 23 | 0 |
| | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4731 | Raw | 72 | 55 | 46 | 38 | 30 | 22 | 0 |
| | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4732 | Raw | 72 | 54 | 47 | 40 | 33 | 27 | 0 |
| | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4733 | Raw | 72 | 57 | 49 | 41 | 33 | 26 | 0 |
| | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4734 | Raw | 72 | 55 | 48 | 41 | 34 | 27 | 0 |
| | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4735 | Raw | 72 | 52 | 45 | 38 | 32 | 26 | 0 |
| | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4736 | Raw | 72 | 57 | 50 | 44 | 38 | 32 | 0 |
| | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |
| 4737 | Raw | 72 | 52 | 46 | 40 | 34 | 29 | 0 |
| | UMS | 100 | 80 | 70 | 60 | 50 | 40 | 0 |

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**Specification Aggregation Results**

Overall threshold marks in UMS (ie after conversion of raw marks to uniform marks)

| | Maximum Mark | A | B | C | D | E | U |
|---|---|---|---|---|---|---|---|
| **3890** | 300 | 240 | 210 | 180 | 150 | 120 | 0 |
| **3891** | 300 | 240 | 210 | 180 | 150 | 120 | 0 |
| **3892** | 300 | 240 | 210 | 180 | 150 | 120 | 0 |
| **7890** | 600 | 480 | 420 | 360 | 300 | 240 | 0 |
| **7891** | 600 | 480 | 420 | 360 | 300 | 240 | 0 |
| **7892** | 600 | 480 | 420 | 360 | 300 | 240 | 0 |

The cumulative percentage of candidates awarded each grade was as follows:

| | A | B | C | D | E | U | Total Number of Candidates |
|---|---|---|---|---|---|---|---|
| **3890** | 37.64 | 54.75 | 68.85 | 80.19 | 88.46 | 100 | 18954 |
| **3892** | 58.92 | 74.42 | 85.06 | 91.87 | 96.04 | 100 | 2560 |
| **7890** | 47.57 | 68.42 | 83.78 | 93.17 | 98.15 | 100 | 11794 |
| **7892** | 60.58 | 80.66 | 90.76 | 95.89 | 98.72 | 100 | 2006 |

For a description of how UMS marks are calculated see:
http://www.ocr.org.uk/learners/ums_results.html

Statistics are correct at the time of publication.

**List of abbreviations**

Below is a list of commonly used mark scheme abbreviations. The list is not exhaustive.

| | |
|---|---|
| AEF | Any equivalent form of answer or result is equally acceptable |
| AG | Answer given (working leading to the result must be valid) |
| CAO | Correct answer only |
| ISW | Ignore subsequent working |
| MR | Misread |
| SR | Special ruling |
| SC | Special case |
| ART | Allow rounding or truncating |
| CWO | Correct working only |
| SOI | Seen or implied |
| WWW | Without wrong working |
| Ft or √ | Follow through (allow the A or B mark for work correctly following on from previous incorrect result.) |

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